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Strategies for Economic Ordering of Perishable Goods under Demand-Dependent Final Production Rate and Partially Backordered Conditions

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ABSTRACT

The current study determines the best course of action for perishable goods in the presence of price-dependent demand final production rate, and partial backordering, which is influenced by lot sales and waiting time duration. A shortage in a competitive market serves as a flood of new customers. Accordingly, a portion of the overall deficiency has been improved in each round through the backlog method in order to prevent their discontent with having to wait for an extended period of time. According to the literature on inventory, shortages typically arise after early inventory building. However, the current model has considered a different strategy than the inventory followed by shortage: shortage followed by inventory. A demand-dependent production rate has been included to the new shortage method in addition to the constant production rate used in earlier models. As shown by the tabulated findings, the object was therefore judged to be additional commercial than earlier models. Therefore, the management should control the manufacturing pace of the final goods based on the influx of customers in order to make a business permanently profitable. To get the best results, an algorithm has been applied. Through sensitivity analysis, the model is thoroughly investigated numerically using various values for the various parameters involved.

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INTRODUCTION:

Every item in any inventory has naturally deteriorated with time: this is a natural process. Products like food items and blood banks. The spontaneous deterioration of every item in any inventory over time is a natural occurrence. During routine storage, things such as food items, blood banks, radioactive materials, chemicals, pharmaceuticals, and other items degrade enough. Therefore, while creating the best possible inventory control strategy for these perishable goods, deterioration-related Inventory loss can't be disregarded.

. Consequently, we have included a decline component in your model. Price rises and decreases are observed to be accompanied with fluctuations in the demand for specific commodities in supermarkets. Thus, one variable that is influenced by pricing is demand. Rating and batch sizing choices are critical for perishable things because the price of the product is determined by demand Shortage is another unavoidable situation that periodically occurs in a market. There is an excessive number of customers. Therefore, backlogs are an essential protection against financial loss brought on by the depreciation of unpreserved assets. Perishable commodities and backlogs are consequently complementary. The inventory models established by investigators show that the backlog amount is linearly related to The total number of customers waiting in queue. The backlog rate is also influenced by how long it takes for the following batch to arrive. To gain a contemporary knowledge, we review the works of Chang and Dye [4], Mak [3], Park [2], and Montgomery et al. [1]. This object accepts that the partial ranking is based on the coming up time of the backlog. This is a well option because trades don't want to delay for backlogs. Raafat's inventory models, which address perishable items in scenarios of fixed demand, finite output, and backlog, were cited in this context [6]. Kim and Hwang [7], Rubin et al. [9], Burwell [10], and Hwang et al. [8]. Giri et al. [11] and Jalan and Choudhury [12] investigated the optimal approach for a perishable product that is facing shortages. Sena [13] talked about the best selling price and lot size for time-varying degradation with partial backlog. The majority of the models covered above were developed as cost reduction problems that include lot sale costs and backorders. The existence of the universal cost minimization problem has been demonstrated by Abad [5]. A generic methodology for determining the ideal lot size and maximum back order level has been established by Lue [14]. While the aforementioned models addressed the strategies of inventory followed by shortages, the current model addresses deficiencies followed by inventory, producing a somewhat better outcome. Additionally, as opposed to the fixed production rate seen in many earlier models, we have included a demand-dependent production rate. In addition to being thoroughly presented and optimally solved with Mathematica, the model is also examined through sensitivity analysis with algebraic examples.

ASSUMPTIONS & NOTATIONS:

The resulting notations and presumptions have been utilized for the reproduction's simplicity.

p = Selling price per unit.

D (p) = Demand rate (units / period).

λD = Production rate (units/period).

Hear $\lambda > 1$,

θ = Constant deterioration rate, 0 <θ<1<. η = waiting time of the customer.

B (η) = $\frac{Kg}{1+k_1 n} < 1\forall\eta$ is the fraction of the shortage backlogged,

0 < K_0< 1, 0 K_1< 1.

I (t) = Instant inventory level.

T_0= Stock out time span.

T_1= Shortage recovering time span.

T_2= Time at which the inventory level raises to peak level.

T = Cycle length.

h = Inventory carrying cost per unit period.

v = Unit cost.

Q = Lot-size ordering quantity.

$D = \frac{dD(p)}{dp} < 0, \forall\, p\, \epsilon(0, \infty)$.

Minimal revenue = $\frac{d[pD(p)]}{dP}$ = p + D > 0, strictly rising function of p, hence $\frac{1}{D(p)}$ is a curved function of p.

Over an endless planning horizon, one inventory is considered. It is assumed that lead time is zero. A shortage is acceptable.

Mathematical Formulation :

There are no inventories at the beginning of the cycle, and shortages build up over time. [0, T_0]. In order to satisfy the present demand as well as some of the backlogged request, manufacturing starts at t = T_0. The deficiency has been corrected at time T1, and a higher level of inventory is beginning to accumulate. The inventory level decreases as a result of production ceasing at time T_2. As a result, following cycle T, the inventory level terminates with zero stock. Figure 1 illustrates the inventory cycle's visual behavior.

The following differential equations are considered in order to mathematically resolve the discussion.

$$\frac{dI(t)}{dt} = -DB\ (T_0 - T), \quad 0 \le t \le T_0, I(0) = 0, \quad (1)$$

$$\frac{dI(t)}{dt} = D(\lambda - 1), \quad T_0 \le t \le T_1, I(T_1) = 0, \quad (2)$$

$$\frac{dI(t)}{dt} + \theta I(t) = D(\lambda - 1), \quad T_1 \le t \le T_2, I(T_1) = 0, \quad (3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad T_2 \le t \le T, I(T) = 0, \quad (4)$$

With the specified boundary conditions, we have solved the differential equations (1) to (4) above.

$$dI(t) = -DB(T_0 - t)dt = -\frac{DK_0}{1 + K_1(T_0 - t)}dt$$

$$\therefore I(t) = \frac{DK_0}{K_1}\ln\left(\frac{1 + K_1(T_0 - t)}{1 + K_1T_0}\right), 0 \le t \le T_0, \quad (5)$$

$$I(t) = D(K - 1)(t - T_1), T_0 \le t \le T_1, \quad (6)$$

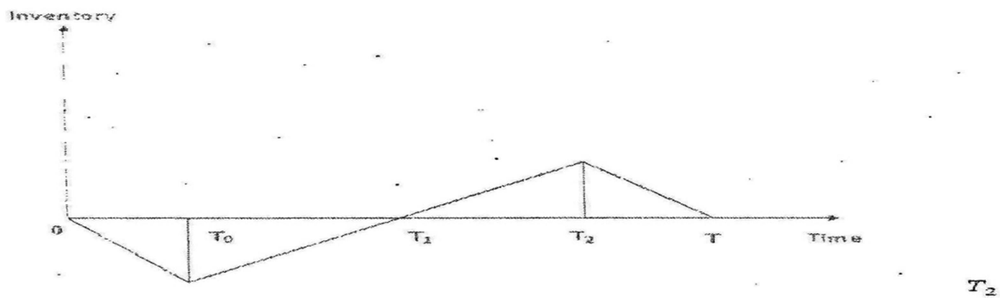
$$I(t) = \frac{D(\lambda - 1)}{\theta}\left[1 - e^{\theta(T_1 - t)}\right], T_1 \le t \le T_2, \quad (7)$$

$$I(t) = \frac{D}{\theta}\left[e^{\theta(T - t)} - 1\right], T_2 \le t \le T. \quad (8)$$

Since the point of intersections of the trajectories of equations (5) and (6) is at t = T_0.

Thus, $-\frac{DK_0}{K_1} ln(1 + K_1T_0) = D(\lambda - 1)(T_0 - T_1)$

$\Rightarrow T_1 = T_0 + \frac{K_0 ln(1 + K_1T_0)}{k_1(\lambda - 1)}$ (9)



Comparing the common prices of equations (7) and (8) at t = T_2, we get

$$\frac{D(\lambda - 1)}{\theta}\left[1 - e^{\theta(T_1 - T_2)}\right] = \frac{D}{\theta}\left[e^{\theta(T - T_2)} - 1\right] (10)$$

On solving, we have

$$T_2 - T_1 = \frac{1}{\theta} ln\left[1 + \frac{1}{\lambda}\left(e^{\theta(T - T_1)} - 1\right)\right] \quad (11)$$

Given that the unit is produced within the time frame

[T_0, T_2], henceforth the lot-size

Q = $\lambda D(T_2 - T_0)$.

The money made from the production's sale at that time [T_0, T] is

pD [$\lambda(T_1 - T_0) + (T - T_1)$].

Since the unit is produced inside the time frame [T_0, T_2], the manufacturing cost of the material quantity is

Qv = Dλv ($T_2 - T_0$) (12)

Given that [T_1, T] is the demand time span in the absence of a shortage and [T2, T1] is the production space when the positive inventory cycle is available. Therefore, the difference between production volumes and demand quantities throughout the positive inventory time period equals the number of degraded units.

Therefore, the cost of deterioration is

$$Nv = vD[\lambda(T_2 - T_1) - (T - T_1)], \quad (13)$$

where N is the quantity of units that have deteriorated.

The holding cost in the gap is because the stock is available during the interval [T_1, T].

$$H = h\int_{T_1}^{T} I(t)dt$$

$$= \frac{hD(\lambda-1)}{\theta}\int_{T_1}^{T_2}\left[1 - e^{\theta(T_1-1)}\right]dt + \frac{hD}{\theta}\int_{T_2}^{T}\left[e^{\theta(T-t)} - 1\right]dt$$

$$= \frac{hD(\lambda-1)}{\theta}\left[T_1 - T_1 + \frac{e^{\theta(T_2-T_2)} - 1}{\theta}\right] - \frac{hD}{\theta}\left[T - T_2 + \frac{1 - e^{\theta(T_2-T_1)}}{\theta}\right]$$

$= \frac{hD}{\theta}[(T - T_1)(T_2 - T_1) - T]$ with the help of equation (10) (14)

The profit of the total cycle [0, T] is F (p, T_0, T) = Selling income – production cost-holding cost-set up cost.

Thus, $F(p, T_0, T) = \frac{D\lambda\ _0(p-v)ln(1+K_1T_0)}{K_1(\lambda-1)} + D\ (T - T_1)\left(p + v + \frac{h}{\theta}\right)$

$$-\lambda D(T_2 - T_1)\left(2v + \frac{h}{\theta}\right) - A \quad (15)$$

Given that [T_1, T] is the demand time span during which there is no shortage and

[T_2, T_1] is the production interval during which the positive inventory cycle is available.

Average profit per cycle is

$$\prod(p, T_0, T) = \frac{F(p, T_0, T)}{T}$$

$$= \left[\frac{D\lambda K_0(p-v)\ln(1+K_1T_0)}{K_1(\lambda-1)} + D\left[T - T_0 - \frac{K_0 ln(1+K_1T_0)}{k_1(\lambda-1)}\right]\left(p+v+\frac{h}{\theta}\right)\right.$$

$$-\lambda D\left[\frac{1}{\theta} ln\left\{1+\frac{1}{\lambda}\left(e^{\theta\left(T-T_0-\frac{K_0 ln(1+K_1T_0)}{k_1(\lambda-1)}\right)}-0\right)\right\}\right]\left(2v+\frac{h}{\theta}\right) - A]/T$$

Hence we have to maximixe$\prod(p, T_0, T)$ under the conditions

$T_0 \geq 0$

$T \geq T_1$

$p \geq v$ where p is a decision variable.

Alogrith for optimization:

When a decision variable is the selling price (p) and $\prod(p, T_0, T)$ is not a pseudoconcave function then the profit function $\prod(p, T_0, T)$ may have many local maxima. Let $\prod(p/T_0, T)$ signifies $\prod(p, T_0, T)$ when T and T_0 are stable. In order to maximize the complicated unrestricted problem $\prod(p, T_0, T)$, Standard non-linear programming software has been utilized. Mathematica. The following is the procedure used to solve the problematic.

Step 1 Let $p = p_i$ where p_i signifies some value of p.

Step 2 For this optimum p, let $T_0 = T_0^*$ and T = T*

Step 3 Let the optimum result $T_0 = T_0^*$ and T = T maximizes F(p/T_0, T) locally. Let the optimum result $T_0 = T_0^*$ and $T = T^*$ maximizes F(p/T_0, T) locally. Let F(p/T_0, T) be maximum for p = p*.

Step 4 Taking this p = p* repeated the step 2 and 3 till $\prod(p, T_0, T)$ achieves local maxima. Until it reaches global maxima, this process has been repeated multiple times with different values of p.

Numerical Example:

Suppose (p) = 1,600,000 p^{-4}, v = \$6.3 / unit, A = \$1000/production run, h = \$1/unit/week, $\theta = 0.2, K_0 = 0.9, K_1 = 0.1$ and $\lambda = 2.4$.

The optimum result was found to be p = 8.49999, T_0 = 7.13, T = 11.002. With help of these solutions we also obtained

$T_1 = 10.59015$, $T_2 = 10.7659$, $Q = 2674.66$, $\prod(p, T_0, T) = 438.45882$.

Table 1 summarizes a comparison of a demand-dependent manufacture rate versus a fixed production rate using the same numerical data.

Table 1 Ideal Value

Symbols	Variable Production rate	Fixed production rate	Remark (Present model)
p	8.499	7.875	Sells at hgiher price
T_0	7.163	7.49939	Production starts earlier
T_1	10.63412	11.0869	Backlog clears earlier
T_2	10.79510	11.20229	Production stops earlier
T	11.010	11.35879	Cycles repeat quickly
Q	2672.59	3702.78	Orders less quantities
N_d	2.56179	1.7915	Deteriorating quantity is more
I_{max}	67.9539	66.59	Production stops at higher inventory
Π	438.513	422.321	About 4% more profit
T_2-T_1	0.16089	0.27101	Inventory available for lesser time
T_1-T_0	3.63310	3.58799	Waiting time of the custome is less
D	306.5110	416.05079	Less demand as price is higher
R	735.62679	1000	Less Production quantity makes more profit

A four percent increase in earnings is no small accomplishment in any company community. In summary, the current article is a better model in every way since it introduces a demand-dependent production rate instead of a set production rate, which differs from earlier models.

Table 2 Sensitivity analysis

	T_0 Production Starts	T_1 shortage ends	T_2 Production ends	T optimal cycle time	P optimal price	Q optimal order quantity	D demand	λD production	profit function
h									

0.5	7.1199	10.5763	10.76121	11.008	8.5009	2678.565	306.50973	735.63337	439.162
1.0	7.1619	10.63411	10.79506	11.010	8.49900	2672.581	306.51117	735.62682	438.511
1.5	7.1998	10.68619	10.81934	11.001	8.43001	2752.10	316.81787	760.36298	438.309
A									
900	7.1619	10.63419	10.79419	11.010	8.49989	2672.581	306.51114	735.62682	438.5139
1000	7.1619	10.63419	10.79419	11.010	8.49989	2672.581	306.51114	735.62682	429.3694
λ									
2.3	6.9929	10.6638	10.83269	11.061	8.50001	2706.867	306.50971	704.97231	444.1401
2.4	7.1619	10.63411	10.79506	11.010	8.49990	2672.581	306.51117	735.62682	438.5139
2.5	7.3409	10.64299	10.78932	10.489	8.47001	2680.029	310.81538	777.03841	433.5255
v									
5.8	7.1600	10.63139	10.78959	11.002	7.75201	3859.531	443.06241	1063.3491	587.8301
6.3	7.1619	10.63319	10.79506	11.010	8.49989	2672.581	306.51117	735.62682	438.5141
6.8	7.1669	10.64100	10.79371	11.001	9.09989	2031.761	233.42391	560.21731	330.3592
K_0									
0.9	7.1619	10.63411	10.79506	11.010	8.49989	2672.581	306.51117	735.62682	438.5141
0.8	7.3989	10.56399	10.79479	11.101	8.51001	2485.700	305.07156	732.17176	395.6101
K_1									
0.1	7.1619	10.63411	10.79506	11.010	8.49989	2672.581	306.51117	735.62682	438.5141
0.11	7.3549	10.82611	10.90395	11.009	7.35981	2618.164	307.81133	738.74721	403.9494
θ									
0.2	7.1619	10.63411	10.79506	11.010	8.49998	2672.581	306.51117	735.62682	438.5143
0.1	6.8791	10.24479	10.72163	10.798	8.49001	2839.788	307.95638	739.09533	440.6470

Uncertainties in scenarios involving decision-making may cause changes in the values of the different parameters involved in any inventory scheme. Sensitivity analysis is essential for examining how the aforementioned modifications affect optimality. The following sources have been selected based on the different outcomes of our model that are shown in the sensitivity analysis table.

The ideal profit and the optimal ordering quantity both rise as the degradation parameter θ does, while the optimal price falls.

Both the ideal profit and the ideal ordering quantity decrease when the backlog is decreased during the stock-out period, but the ideal trade price increases.

Increasing the unit price lowers the selling price while decreasing the ordering quantity and ideal profit.

The selling price, optimal profit, and ordering quantity all decline as production increases.

CONCLUSION :

The current study finds the optimal strategy for perishable commodities in the presence of price-dependent demand, demand-dependent production, and partial backordering based on the dimensions of waiting time and lot sale. Since production is dependent on demand and demand is influenced by selling prices, as well as because production commodities are perishable, demand backlogs are necessary to prevent needless expenses from deteriorating. A shortage in a competitive market serves as a flood of new customers. In this article, a partial backlog of the entire shortfall has been recovered in each cycle to prevent their discontent with waiting for extended periods of time. This is provided by the location of the consumers' waiting time for their merchandise. According to the literature on inventory, shortages typically arise after early inventory building. In contrast to inventory followed by scarcity, the new strategy of shortage followed by inventory is more profitable. There has been a notable increase in profit recorded by the current model. The reason for this is that a variable demand-dependent production policy was implemented instead of a steady production policy. In an unstable business environment, we expect that the current essay will undoubtedly help the global business community regulate output based on the attitudes and purchasing power of customers

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