



Dynamical Behavior and Chaotic Characteristics of Communication with Switching in Wireless Sensor Networks

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ABSTRACT

Data gathering, congestion, data overlapping, collision and data loss etc are very common problem In wireless sensor network (WSN). Congestion in WSN not only lead to packet losses, delay, and but also leads to excessive energy consumption due to a large number of packet drops and retransmissions. At the same time when two or more close nodes are sending data to others then collision occurs. Due to that reason in the network data collision creates Chaos. In this paper, we proposed a nonlinear dynamical model with switching and applied chaos theory to study the important traffic behavior of the WSN and several aspects of the model have been discussed mathematically and found the chaotic behavior when packets collided in WSNs. This is a model where two prey nodes are competing for the unique resource that is why switching behavior is considered in this model with lone predator node and switch to alternative prey node. For the dead lock situation we investigate the responsible parameters from the model through simulation. We find chaotic situation in WSN on embedded dimensions and little variation of the sensible parameters with different topologies of WSN creates chaos. Heuristic simulation carried out and observed important parameters for stability and instability situations of the model. We evaluate the proposed model using Matlab and discussed the effectiveness of the model with the simulation

experiments.

1. INTRODUCTION

The proper data transport is very much important issue in WSNs where different applications have different needs. Wireless sensor network is facing various problems such as congestion, data overlapping and data loss etc. The characteristic of sensors are limited power supply, processing ability and less memory which is very much challenging at transport protocol. Therefore, it is very much challenging task in Sensor Networks to delivery authentic data between sink nodes and sensor. Mainly there are for two reasons for congestion in wireless sensor network when same channel is using for multiple data transmission at a time or when the the received data fails to forward to the net routing node. When congestion occur in a Wireless Sensor Network that cause packet or data loss and packet loss which affects on throughput of the system and packet loss is very much effective for energy consumption and question of reliability of end nodes. For designing any protocol related to transfer data the energy efficient protocol should be considered. At the same time when more than one nodes are attempted to send data to others then collision occurs. Due to that reason in the network data collision creates Chaos. We consider a mathematical model to avoid the dead lock situation which can exhibit only basic pattern such as equilibrium, a limit cycle or a chaos. For certain parametric choices the effect of several variables in the system the chaotic dynamics may occur. Sensitive dependence on initial conditions are another key feature of chaotic dynamic that is why system may exhibits chaos with small change in the initial conditions. For a different values of parameter for long time behavior may shows chaos. The local stability based on sensor node receives and transmits for the system have been obtained and analyzed. The proposed model can gather sensor information with low energy consumption and exhibits various chaotic scenario. We evaluate the proposed model using Matlab and discussed the effectiveness of the model with the simulation experiments.

2. RELATED WORK

At transport layer that provide at the same time the when more than one nodes are sending data to others then collision occurs. Due to that reason in the network data collision creates Chaos. To avoid collisions many CSMA based MAC protocols are proposed in WSNs, such as in [1] and [2]. Due to collisions applications and loss of information may fail from the base stations. Several proposals were done in the past like [3] or [4] that shows the use of chaotic systems in cryptography is not new and several cryptanalysis like [5] or [6] were also published. But here in our work we propose a nonlinear model with

switching. Other previous works could also be referred which was without switching. Nevertheless a new research effort to the cryptology community on the subject is proposed in [6]. There is yet a great field of unsolved issues concerning its security as stated for example in [7]. The small dimension and the common exposure and low profiles of processing, storage and energy consumption the security issues on WSN all around the world. WSN is presented in several aspects in [8] with a classification of Homogeneous WSNs and Heterogeneous WSNs. This paper discuss about data gathering and collision with switching which are characterized by the fact that all the network nodes have the same properties. In this paper, we proposed a nonlinear dynamical model with switching and applied chaos theory to study the behavior of the WSN. Several aspects of the model have been discussed mathematically and found the chaotic behavior when collision occur in WSNs.

3. PROBLEM DEFINITATION AND SYSTEM MATHEMATICAL MODEL

At transport layer that provide at the same time the when more than one nodes are sending data to others then collision occurs. Due to that reason in the network data collision creates Chaos. Due to collision packet loss happened and simultaneously degrade the performance of the network. Considering all asoects with data collision, data transfer and loss together forms the system of differential equations.

Therefor the model consisting the system of equations are given below:

$$\begin{aligned}\frac{dX_1}{dt} &= r_1 X_1 \left(1 - \frac{X_1}{K_1} - \frac{c_{12} X_2}{K_1}\right) - f_1 F_1(X_1, X_2) X_3 \\ \frac{dX_2}{dt} &= r_2 X_2 \left(1 - \frac{X_2}{K_2} - \frac{c_{21} X_1}{K_2}\right) - f_2 F_2(X_1, X_2) X_3\end{aligned}\tag{1}$$

$$\frac{dX_3}{dt} = e_1 f_1 F_1(X_1, X_2) X_3 + e_2 f_2 F_2(X_1, X_2) X_3 - dX_3$$

$$\text{where } f_1 = \frac{\pi X_1}{\pi X_1 + (1-\pi)X_2} \text{ and } f_2 = \frac{(1-\pi)X_2}{\pi X_1 + (1-\pi)X_2}$$

where π is the prey preference and takes a value between 0 and 1.

$$F_i(X_1, X_2) = \frac{A_i X_i}{(1 + B_1 X_1 + B_2 X_2)}; \quad i=1,2$$

where from prey node X_i to predator node the maximum harvest rate is A_i and half-saturation constant is $1/B_i$. Where conversion rates from X_i to X_3 are e_1 and e_2 . Both f_1 and f_2 are fixing to 1 in the system, once



the system considering without switching. We have found chaos in our model without switching in another paper. We carry out detailed investigations of chaos in the system with and without switching as to know whether the chaotic dynamics, which exists in the model can be observed in the laboratory as well as in natural settings.

Linear Stability Analysis of 3D Model

The system of equations reduces to

$$\begin{aligned}\frac{dX_1}{dt} &= S_1(X_1, X_2, X_3) \\ \frac{dX_2}{dt} &= S_2(X_1, X_2, X_3) \\ \frac{dX_3}{dt} &= S_3(X_1, X_2, X_3)\end{aligned}\quad (2)$$

The criteria for the stability and the existence can be studied by considering

$$\frac{dX_1}{dt} = \frac{dX_2}{dt} = \frac{dX_3}{dt} = 0$$

in equation (2) and solving system (2) it has been found the following seven non-negative equilibria.

F_0 where $X_1 = 0$, $X_2 = 0$, and $X_3 = 0$ is considered at the origin in the dimension space $R^3(x, y, z)$

F_1 is given by $X_1 = K_1$, $X_2 = 0$, $X_3 = 0$

F_2 is given by $X_1 = 0$, $X_2 = K_2$, $X_3 = 0$

F_3 is given by $X_1 = \frac{K_1 - K_2}{V}$, $X_2 = \frac{K_2 - K_1 c_{12} c_{21}}{V}$, $X_3 = 0$ where $V = (1 - c_{12} c_{21})$

Existence of F_4 : Let $X_2 = 0$, X_1 and X_3 are positive solution of

$$r_1 \left(1 - \frac{X_1}{K_1} \right) \frac{A_1 X_3}{1 + B_1 X_1} = 0 \text{ and } \frac{e_1 A_1 X_1}{1 + B_1 X_1} - d = 0 \quad (3)$$

The equilibrium point (X_1^*, X_3^*) is the intersection of two isoclines. Eliminating X_3 from (3) we have



$$X_1^* = \frac{d}{(e_1 A_1 - B_1 d)}, \quad X_2^* = \frac{r_1 e_1 (e_1 K_1 A_1 - B_1 K_1 d)}{(e_1 A_1 - B_1 d)}$$

Existence of F_5 :

Let $X_1 = 0$, X_2 and X_3 are positive solution

$$\text{which gives } X_2 = \frac{d}{(e_2 A_2 - B_2 d)} = \frac{d}{L} \text{ where } L = (e_2 A_2 - B_2 d) \text{ and } X_3 = \frac{r_2 e_2 (e_2 A_2 K_2 - B_2 K_2 d - d)}{K_2 (e_2 A_2 - B_2 d)^2}$$

The intersection of these two isoclines is the Equilibrium point (X_2^*, X_3^*) and which is given by

$$X_2 = \frac{d}{(e_2 A_2 - B_2 d)} \text{ and } X_3 = \frac{r_2 e_2 (e_2 A_2 K_2 - B_2 K_2 d - d)}{K_2 (e_2 A_2 - B_2 d)^2}$$

Existence of F_6 : The existence of F_6 can be studied by investigating the local and global stability. So the community matrix which is obtained from the system is given by

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} = \begin{bmatrix} \frac{\partial S_1}{\partial X_1} & \frac{\partial S_1}{\partial X_2} & \frac{\partial S_1}{\partial X_3} \\ \frac{\partial S_2}{\partial X_1} & \frac{\partial S_2}{\partial X_2} & \frac{\partial S_2}{\partial X_3} \\ \frac{\partial S_3}{\partial X_1} & \frac{\partial S_3}{\partial X_2} & \frac{\partial S_3}{\partial X_3} \end{bmatrix}$$

where The eigenvalues of this matrix are the roots of the equation

$$\lambda^3 + Z_1 \lambda^2 + Z_2 \lambda + Z_3 = 0 \quad \text{Where } Z_1 = [q_{11} + q_{12} + q_{13}],$$

$$Z_2 = q_{22} q_{33} - q_{23} q_{32} + q_{11} q_{22} + q_{11} q_{33} - q_{21} q_{12} - q_{31} q_{13},$$

$$Z_3 = -[q_{11}(q_{22} q_{33} - q_{23} q_{32}) - q_{21}(q_{12} q_{33} - q_{13} q_{32}) + q_{31}(q_{12} q_{23} - q_{13} q_{22})]$$

For F_6 to locally asymptotically stable, we require that $|\lambda| < 1$. From the Routh-Hurwitz criteria, we obtain the required condition as

$$Z_1 > 0, \quad Z_2 > 0, \quad Z_3 > 0, \quad Z_1 Z_2 > 0 \quad (4)$$

4. SIMULATION AND RESULTS

Here most sensitive parameters are investigating through the simulation done by Matlab for the system **with switching** and results are shown below with different parametric values of the model.

$r_1 = 1.6, r_2 = 1.65, K_1 = 150, K_2 = 155, A_1 = 1.6, A_2 = 1.30, c_{12} = 0.204, c_{21} = 0.504, d = 22, B_1 = 0.000054,$
 $B_2 = 0.000055, e_1 = 0.66, e_2 = 2.7.$

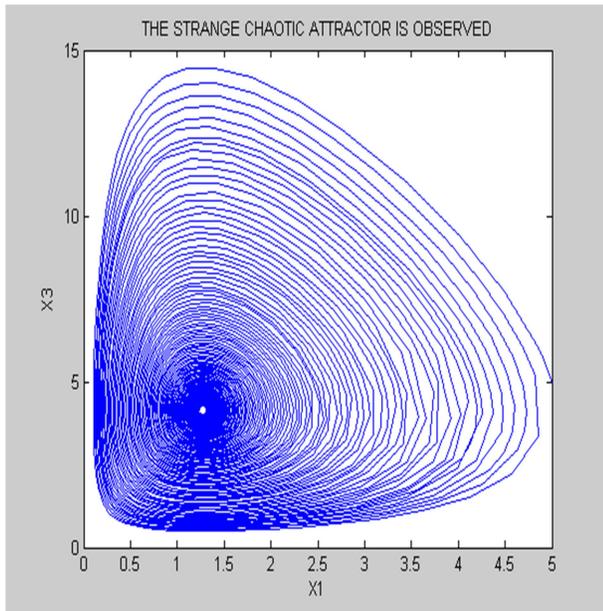


Fig 1: Chaos occur for some sensitive parameters

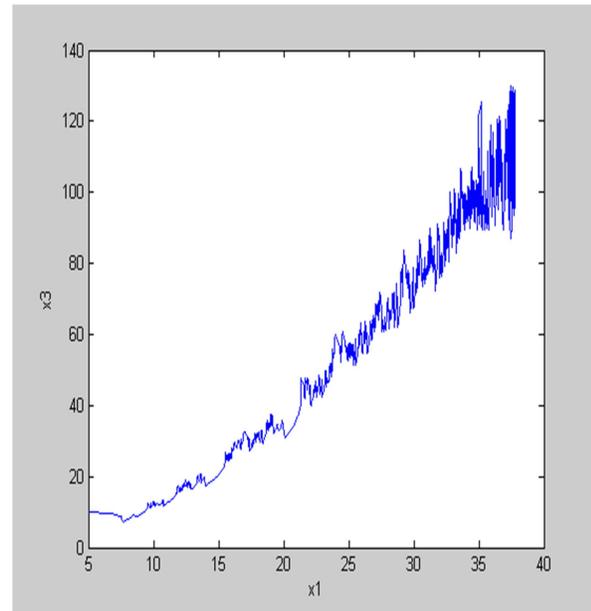


Fig 2: Interaction does not shows chaos

Fig 2 and fig 4 does not show chaos when X_3 and X_2 interact with each other and fig 3 does not show chaos when X_3 and X_1 interact with each other

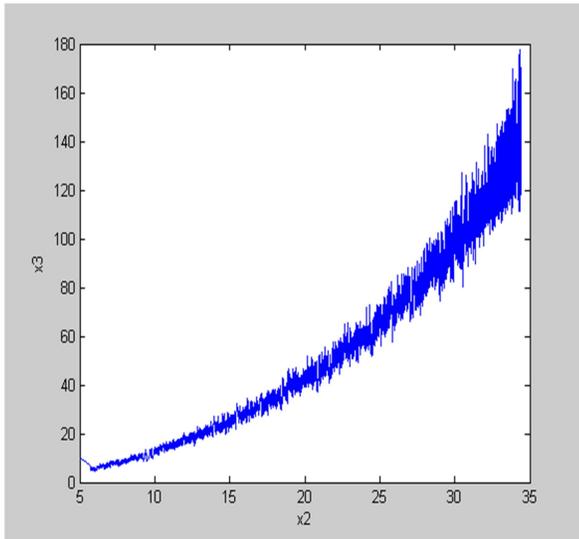


Fig 3: Interaction does not shows chaos

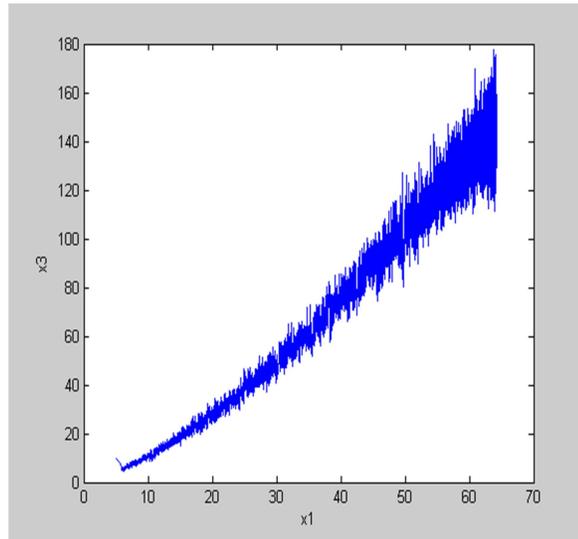


Fig 4: Interaction does not shows chaos

Fig 5 shows the trend of chaotic attractor when X_3 and X_2 interact with each other and fig 6 shows the trend of chaotic attractor when X_3 and X_1 interact with each other

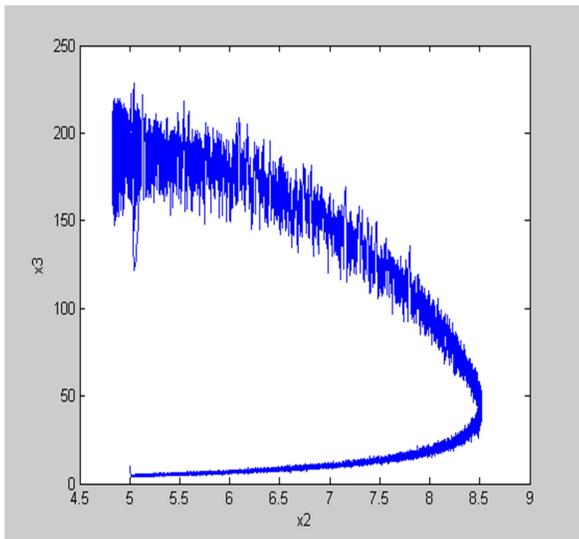


Fig 5: Interaction shows the trend of chaotic

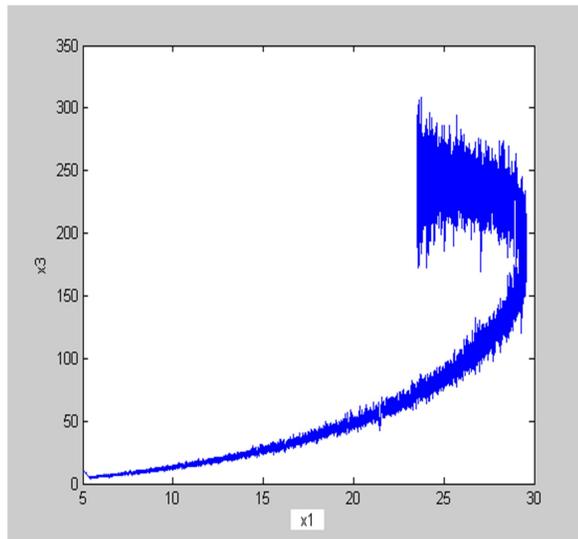


Fig 6: Interaction shows the trend of chaotic

Suitable choice of the parameters satisfying equation (1) and (2) will provide stable equilibrium point and choice of different parameter shows limit cycle by the subsystem with the kolmogorov condition.

CONCLUSION



We have proposed the mathematical model in WSNs with switching. Experimentally some of the features are observed in the network. To determine the regions with effective parameters the computational experiments were done which shows different dynamical behaviors for the system which we have considered. When keeping all the other parameters constants and varying one particular parameter in its different range, thereby fixing the regimes in which the system exhibit chaotic dynamics. We have showed different conditions for stability and persistent criteria for the model. We focus on how to prevent the chaos even if data collisions occur. We investigate the sensible parameters for the chaotic situation of the model through simulation. If the kolmogrov condition satisfied then the subsystem shows limit cycle. Here fig 1 exhibit chaos of our system for the sensitive parameters. Fig 2, fig 3 and fig 4 does not exhibits chaos of the system. But fig 5 and fig 6 have a treand of chaotic nature of the system. The condition (4) gives the stability of the system. This implies that our system with switching gives better solution of the wireless sensor network (WSN) where data gathering, congestion, data overlapping and collision are the common factors.

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