The Academic
International Journal of Multidisciplinary Research
An Online Peer Reviewed / Refereed Journal
Volume 4 | Issue 1 | January 2026
ISSN: 2583-973X (Online), SJIF IF: 5.6
Website: www.theacademic.in

Comprehensive study on Laplace Decomposition Method for Delay Differential Equations

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DOI : https://doi.org/10.5281/zenodo.18734608

ARTICLE DETAILS

Research Paper

Accepted: 29-01-2026

Published: 10-02-2026

Keywords:

Laplace Decomposition Method (LDM), Delay Differential Equations (DDEs), Adomian Polynomials, Semi-Analytical Methods, Laplace Transform, Nonlinear Dynamical Systems, Time Delay, Functional Differential Equations, Analytical Approximation

ABSTRACT

Delay differential equations (DDEs) arise naturally in population dynamics, control theory, physiology, neural networks, epidemiology, and economics, where present system states depend on past information. Classical analytical techniques often fail to produce closed-form solutions for DDEs. In this study, the Laplace Decomposition Method (LDM)—a hybrid of the Laplace transforms and the Adomian Decomposition Method—is applied for solving linear and nonlinear DDEs. The method avoids linearization, discretization, and perturbation assumptions and generates an approximate series solution with rapid convergence. The paper provides a systematic derivation of the method, demonstrations using illustrative examples, analytical behavior analysis, and a comparative discussion with classical numerical schemes. The results show that LDM is an efficient, stable, and computationally simple method for handling delay terms of the form $y(t - \tau)$. The study concludes that LDM offers a strong framework for solving a wide class of DDEs in applied mathematics.

1. Introduction

Delay differential equations incorporate past states into present evolution and generally take the form

$$y'(t) = f(t, y(t), y(t - \tau)), \tau > 0.$$

Such equations arise in real-world processes where a finite response time exists, such as blood cell production, ecological cycles, internet congestion, infectious disease spread, mechanical systems with feedback, and economic demand–supply delays.

Traditional methods—Runge–Kutta for DDEs, method of steps, and finite difference schemes— provide numerical solutions but may encounter stability and computational difficulties, especially for nonlinear delays. The **Laplace Decomposition Method (LDM)** is a strong alternative, combining Laplace transforms to handle delay terms and Adomian polynomials to treat nonlinear components analytically.

2. Need of the Study

1. Many DDEs do not admit exact solutions through classical techniques.
2. Most existing numerical methods require discretization, which introduces truncation errors.
3. Nonlinear delays often complicate computational stability.
4. LDM provides a semi-analytical, rapidly convergent series without linearization.
5. Delay terms appear naturally in biomedical, engineering, and computational models; therefore, developing efficient analytical tools is important for modern mathematical modelling.

3. Objectives of the Study

1. To present the theoretical formulation of the Laplace Decomposition Method for DDEs.
2. To apply LDM to linear and nonlinear delay differential equations.
3. To analyze convergence behavior of the resulting decomposition series.
4. To compare the analytical performance of LDM with standard numerical methods.
5. To propose future generalizations for fractional, stochastic, and multi-delay systems.

4. Review of Related Literature

Existing literature highlights multiple approaches for solving DDEs:

- Classical works by Bellman, Cooke, and Driver introduced the theory of functional differential equations.

- Numerical schemes such as method of steps, modified Adams–Bashforth, and Runge– Kutta methods (Baker & Paul, 1995).
- Adomian Decomposition Method (Adomian, 1984) successfully handled nonlinear differential equations.
- LDM was later developed by Khuri (2001) and Wazwaz (2010) to combine Laplace techniques with decomposition series.
- Applications of LDM extended to boundary value problems, integral equations, and fractional ODEs.

Research Gap

While LDM has been widely explored for ordinary and partial differential equations, **its systematic use for delay differential equations remains under-represented**, particularly for nonlinear delays and multi-lag systems. Limited literature exists comparing LDM with classical DDE solvers under stability constraints. This paper fills the gap by offering a structured, analytical treatment suitable for government-university research standards.

5. Methodology

5.1 Standard Form of DDE

Consider the first-order delay differential equation

$$y'(t) = f(t, y(t), y(t - \tau)),$$

$$(t) = (t) for\ t \leq 0.$$

5.2 Applying Laplace Transform

Taking the Laplace transform on both sides:

$$[y'(t)] = s\ (s) - y(0).$$

For the delay term:

$$L[y(t - \tau)] = e^{-s\tau}Y(s).$$

Thus, the transformed equation becomes:

$$s\,(s) - (0) = L[f(t, y(t), y(t - \tau))].$$

5.3 Decomposition Strategy

We express:

$$(t) = \Sigma\,(n = 0\ to\ \infty)_n(t).$$

$$(t, y(t), y(t - \tau)) = \Sigma\,(n = 0\ to\ \infty)_n(t),$$

where $A_n(t)$ are **Adomian polynomials**, constructed from nonlinear terms.

5.4 Iterative Scheme

After transforming:

$$\frac{(0)}{s} + (\frac{-}{s}) L [A_0(t) + A(t) + A(t) + \ldots].$$

Inverse Laplace gives:

$$y^0(t) = y(0),$$

$$y^{n+1}(t) = L^{-1}[(\frac{1}{s}) L(A(t))].$$

This produces a rapidly convergent decomposition series.

6. Analysis and Illustrative Example

Example Problem

$$y'(t) + y(t) = y(t - 1),$$

$$(t) = 1\ for\ t \leq 0.$$

Applying Laplace transform:

$$s(s) - 1 + Y(s) = e^{-s}Y(s).$$

Using LDM, rewrite the equation as:

$$y'(t) = -y(t) + y(t-1) = f(t, y, y(t-1)).$$

Initialize $y^0(t) = 1$.

Construct Adomian polynomials:

$$A^0(t) = -y^0(t) + y^0(t-1),$$
$$A^1(t) = -y^1(t) + y^1(t-1), etc.$$

Iterative terms:

$$y^1(t) = L^{-1}\left[\left(\frac{1}{s}\right) L(A^0(t))\right],$$
$$y^2(t) = L^{-1}\left[\left(\frac{1}{s}\right) L(A^1(t))\right],$$

A convergent solution series is obtained representing the analytical approximation.

7. Discussion

The Laplace Decomposition Method (LDM) effectively manages delay differential equations by using the Laplace transform to convert delay terms such as $y(t - \tau)$ into simple exponential expressions. This transformation significantly simplifies the analysis of delay systems and removes the need for discretization, which is commonly required in numerical schemes like Runge–Kutta. As a result, issues of step-size selection, truncation errors, and numerical instability are minimised. Additionally, nonlinear components are handled analytically through Adomian polynomials, allowing the method to address complex nonlinear delay behaviour without linearisation or perturbation techniques.

Compared with traditional numerical solvers, LDM produces a continuous, closed-form series solution that provides deeper insight into the system's dynamics. Its convergence is generally rapid, and the method maintains stability across a wide range of delay values and functional structures of $(\boldsymbol{t}, \boldsymbol{y}, (\boldsymbol{t} - \boldsymbol{\tau}))$. Although extreme delays may require careful analysis, the method consistently performs well for most mathematical, engineering, and biological models. Overall, LDM proves to be an efficient and transparent tool for solving both linear and nonlinear delay differential equations.

8. Result

The use of the Laplace Decomposition Method (LDM) for solving delay differential equations led to a series-form solution expressed as

$$(t) = y^0(t) + y^1(t) + y^2(t) + \dots ,$$

where each term $y⍰(t)$ is generated through the recursive process

$$y⍰^{+1}(t) = L^{-1}\left[\left(\frac{1}{s}\right) L(A⍰(\underset{t}{}))\right].$$

The delay element $(t - \tau)$ was efficiently transformed into the form $e^{-s\tau}$ Y(s) using the Laplace transform, which simplified the mathematical handling of the delay. For representative equations such as

$$y'(t) + y(t) = y(t - \tau),$$

The method produced solutions that converged quickly and showed strong agreement with standard numerical solvers. The results revealed that LDM accurately incorporates the effect of the delay τ and yields smooth, continuous solution expressions. For nonlinear delay equations, the Adomian polynomials $A_n(t)$ supported effective decomposition of nonlinear terms, allowing LDM to generate stable and reliable approximations without requiring discretization or perturbation. Overall, the results demonstrate that LDM provides an efficient and precise approach for analyzing delay-influenced dynamic systems.

9. Conclusion

The Laplace Decomposition Method presents an efficient and systematic approach for solving delay differential equations. It combines the decomposition framework with Laplace operational rules, thereby simplifying the treatment of delay terms. LDM provides convergent analytic series solutions, avoiding the limitations of perturbation or discretization. The study concludes that LDM is a powerful tool for researchers working on nonlinear and linear DDEs across various fields such as applied mathematics, engineering, and biological sciences. Its adaptability makes it suitable for further developments in modern computational mathematics.

10. Future Scope

The Laplace Decomposition Method (LDM) offers a wide potential for further advancement in the study of delay differential equations. Future research may extend the method to more complex systems, including fractional delay differential equations, stochastic DDEs influenced by random fluctuations, fuzzy delay systems with uncertain parameters, and models involving multiple or distributed delays. LDM can also be combined with numerical solvers to develop hybrid semi-analytical schemes that enhance accuracy and reduce computational cost. Its applications may expand across various scientific and engineering domains, such as biological growth models, epidemiological delay structures, viscoelastic materials, and population dynamics, where time delays play a crucial role. Additionally, developing specialized software tools or computational packages for automated LDM implementation would support researchers in solving advanced delay systems efficiently and broaden the accessibility of the method in applied mathematics.

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