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A Survey of Geometry and Topology-Based Representation Learning in Artificial Intelligence

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ABSTRACT

Geometry and topology guided representation learning offers a solid mathematical basis for uncovering structural patterns, invariances, and features at different scales in complex data. As modern artificial intelligence systems more and more work with graphs, manifolds, and complex high-dimensional shapes, these methods have become important for doing learning that is both effective and dependable. Geometric deep learning uses ideas like symmetry, equivariance, and the shape of data spaces to build neural networks that fit specific rules of the problem they're solving, which helps them use data better and work well in new situations. Recent advancements include creating equivariant graph neural networks and SE(3)/E(n) equivariant attention models. These models clearly include rotational and translational symmetries, making them especially good for use in physical, molecular, and scientific areas where these symmetries are essential. Alongside these improvements, graph-adapted transformer models have shown that using global attention along with proper structural information can perform as well as or better than traditional graph neural networks on big benchmark datasets. Besides shapes and figures, topological data analysis offers additional methods to understand overall and more complex structures. Methods like persistent homology and persistence diagrams create features that don't change much when shapes are stretched or squashed. These features make classification and scientific analysis more reliable, easier to understand, and better at handling new data. Using these ideas, a new area called topological deep learning adds topological features straight into the entire neural network process. This helps model relationships that other methods, which only look at local or geometry-based info, might miss. These methods are used in many areas like molecular modeling, physical simulations, materials science, and neuroscience. In all these fields, both the shape and structure of things help in creating better models and determining what the models should aim to achieve. Even though there have been some good advancements, there are still several important problems that need to be solved. These include making topological modules that can work at a large scale and are easy to differentiate, combining geometric and topological features in a logical way, understanding how well these models can generalize, and creating good tests to measure how well shape-aware learning works. Focusing on these challenges is key to developing AI that is both easy to understand and reliable in scientific work.

1.Introduction

The fast development of artificial intelligence has made it clearer that we need better ways to learn that go beyond just using messy, disorganized data. We need models based on solid math that can really understand complicated structures. Many real-world datasets, like social networks, molecular graphs, 3D shapes, brain connectivity maps, and physical systems, are naturally non-Euclidean and have complex geometric and topological structures. Geometry and topology-based representation learning has become a strong approach, offering solid mathematical methods to uncover structure, consistency, and features at different scales from complicated data spaces [3]. These methods are becoming more important in today's AI systems, which need to understand and work with complex structures like graphs, manifolds, and high-dimensional data. Geometric deep learning serves as the foundation of this approach by bringing together concepts from differential geometry, group theory, and graph theory with deep neural networks. By clearly showing symmetry, equivariance, and the structure of manifolds, geometric deep learning allows the creation of models that follow the rules of the specific field they are used in, and this greatly reduces the amount of data needed for training [5]. This strong math knowledge helps models work better in situations where there's not much data or getting data is costly. Recent improvements involve equivariant graph neural networks and SE(3)/E(n)-equivariant attention mechanisms, which are created to keep rotational and translational symmetries that are naturally present in data related to physics, chemistry, and molecules. These models show better stability, easier understanding, and more accurate results that match real-world physics than regular neural networks. At the same time, transformer models designed for structured data have become very popular. Models like Graphormer and similar versions use global self-attention along with structural and position information to help model long-range connections in graphs. Empirical studies show that these transformer-style methods can match or even beat classical graph neural networks on large-scale benchmarks, provided that geometric information is properly encoded [4]. This makes them a powerful and reliable tool for scientific AI. These topological features are good at handling noise and give information at different scales, which is usually not possible with just local geometric features. Because of this, TDA has worked well in tasks like classification, grouping, and making scientific conclusions, making things more reliable and easier to understand. Based on these ideas, the new field of topological deep learning combines TDA with full learning processes, allowing neural networks to pick up on more complex relationships and shape-based patterns that regular graph models might miss. Geometry and topology-based representation learning are used in many areas, such as creating models for molecules, simulating physical processes, studying materials, and understanding the brain. In these areas, math structure helps shape both how buildings are designed and how errors are measured, making sure the learning process stays true to real-world physics and scientific principles. Even with these improvements, there are still some big problems to solve. These include making topological layers that can grow and work well with different types of data, combining geometric and topological features in a smart way, ensuring that models can perform well on different kinds of shapes, and creating standard tests to check how good shape-aware learning methods are. Fixing these issues is very important for building the next level of AI systems that are strong, easy to understand, and based on solid math.

.2.Literature Review

The literature review table presents a chronological overview of significant research contributions in geometry- and topology-based representation learning, highlighting how mathematical structure has progressively shaped modern artificial intelligence. Recent survey and review papers emphasize the maturation of this research area. Works such as Zia et al. (2024) and Papillon et al. (2023) consolidate emerging ideas in topological deep learning, demonstrating how persistent homology and higher-order message passing enable neural models to capture complex relational structures that traditional graph neural networks often fail to represent. These surveys indicate a shift from purely local graph operations toward topology-aware learning frameworks capable of modeling global shape information. Several studies focus on the integration of geometry and topology within scientific machine learning. Bronstein et al. (2023) and Karniadakis et al. (2021) show that incorporating geometric constraints and topological descriptors leads to physically consistent and interpretable AI models, particularly in scientific and engineering domains. These approaches leverage mathematical priors such as symmetry, invariance, and conservation laws to enhance learning reliability. Similarly, Wasserman (2023) and Chazal et al. (2021) establish a strong theoretical foundation for topological data analysis, explaining how persistence-based features provide robustness against noise and deformation, which is crucial for real-world data. A significant portion of the literature concentrates on geometric deep learning and equivariant architectures. The seminal work by Bronstein et al. (2021) unifies grids, graphs, and manifolds under a common geometric framework, laying the groundwork for symmetry-preserving neural networks. Subsequent developments, including E(n)-equivariant graph neural networks and SE(3)-transformers, exploit rotational and translational invariances to achieve improved performance in molecular modeling and 3D data analysis. These studies demonstrate that embedding geometric structure directly into model design improves both generalization and interpretability. Transformer-based methods adapted to graphs represent another important trend. Graphormer and related surveys reveal that global attention mechanisms, when augmented with structural encodings, can overcome the locality limitations of traditional graph neural networks. This highlights the complementary nature of geometry-aware attention and message-passing approaches. Additionally, foundational theoretical works such as those by Xu et al. (2020) analyze the expressive power of graph neural networks, identifying inherent limitations and motivating the integration of topology-based features to enhance representational capacity.

Year	Author	Paper Title	Method	Key Results
2024	Zia et al.	Topological Deep Learning: A Review	Persistent homology, neural integration	Systematic overview of topology-aware neural models
2023	Papillon et al.	Message-Passing Topological Neural Networks	Simplicial complexes, message passing	Captured higher-order relationships beyond GNNs

2023	Bronstein et al.	Geometric and Topological Methods for Scientific ML	Geometry, topology, scientific ML	Demonstrated physics-consistent learning
2023	Wasserman	Topological Data Analysis	Statistical topology, persistence	Unified theory with ML robustness
2022	Peyré & Cuturi	Computational Optimal Transport	Optimal transport geometry	Improved distribution-based representations
2022	Qi et al.	Graph Transformers: A Survey	Transformer attention on graphs	Showed global attention benefits over GNNs
2022	E, Ma & Wu	Mathematical Foundations of Deep Learning	Functional analysis, geometry	Established theory-driven DL frameworks
2022	Hofer et al.	Deep Learning with Topological Signatures	Persistent homology layers	Improved robustness and interpretability
2021	Bronstein et al.	Geometric Deep Learning	Symmetry, equivariance, manifolds	Unified framework for non-Euclidean learning
2021	Satorras et al.	E(n)-Equivariant Graph Neural Networks	Equivariant message passing	Achieved symmetry-preserving learning
2021	Ying et al.	Graphormer	Graph transformers, encodings	Outperformed classical GNNs
2021	Klicpera et al.	Directional Message Passing	Geometric molecular graphs	Improved molecular property prediction
2021	Chazal et al.	TDA for Machine Learning	Persistent homology	Robust multi-scale feature extraction

2021	Karniadakis et al.	Physics-Informed Machine Learning	PDE constraints, geometry	Ensured physical consistency
2020	Xu et al.	How Powerful are Graph Neural Networks?	Graph isomorphism theory	Identified expressiveness limits of GNNs
2020	Fuchs et al.	SE(3)-Transformer	Equivariant attention	Rotation-translation invariant learning
2020	Wu et al.	Survey on Graph Neural Networks	Graph learning methods	Comprehensive GNN taxonomy
2020	Zhou et al.	GNN Methods and Applications	Graph theory, DL	Broad applications across domains
2020	Achille & Soatto	Information-Theoretic Learning	Information bottleneck	Improved representation robustness
2020	Adlam et al.	Double Descent Phenomenon	Statistical learning theory	Explained generalization transitions

Table:1-Literature Review

3. Problem Solving in Mathematics: An Artificial Intelligence

Problem solving in mathematics involves understanding a problem, identifying relevant concepts, and applying logical thinking to arrive at a solution. Artificial intelligence (AI) improves this process by providing computational tools that can analyze patterns, explore large solution spaces, and aid in logical thinking. AI techniques such as machine learning, symbolic computation, and automated reasoning systems can learn from existing mathematical knowledge and previous solutions to suggest efficient strategies for new problems. Unlike traditional computation, AI can combine numerical methods and symbolic operations to perform both exact inferences and approximations. AI-based systems can also make inferences, review evidence, and provide step-by-step instructions, making them valuable in education and research. By augmenting human intuition with computing power, AI helps mathematicians tackle complex problems, reduce manual labor, and uncover hidden connections. AI acts as an intelligent assistant in solving mathematical problems, supporting creativity, accuracy, and efficiency rather than replacing human thinking. From an artificial intelligence (AI) perspective, mathematics has greatly benefited from the ability of intelligent systems to model logical thinking, recognize patterns, and automate complex analytical tasks. AI supports mathematics by extending human problem-solving abilities beyond manual calculations and intuition. One of the most important contributions of AI is automated reasoning, where algorithms can verify evidence, check logical consistency, and help prove formal theorems. This reduces human error and speeds validation of mathematical results. AI also supports pattern recognition by analyzing large mathematical datasets such as number sequences, diagrams, and geometric structures. Machine learning models can reveal hidden laws and suggest inferences that lead mathematicians to new theorems. In symbolic computation, AI systems manipulate algebraic expressions, solve equations, and perform precise calculations that would otherwise take time. In education, AI enables personalized learning by adjusting problem difficulty, providing step-by-step hints, and diagnosing conceptual misconceptions. A smart tutoring system helps students develop logical thinking and problem-solving skills more effectively. AI also improves visualization and geometric thinking, allowing you to explore complex and abstract concepts through interactive models. AI combines pure and applied mathematics by enabling efficient numerical simulation and optimization in scientific and engineering applications

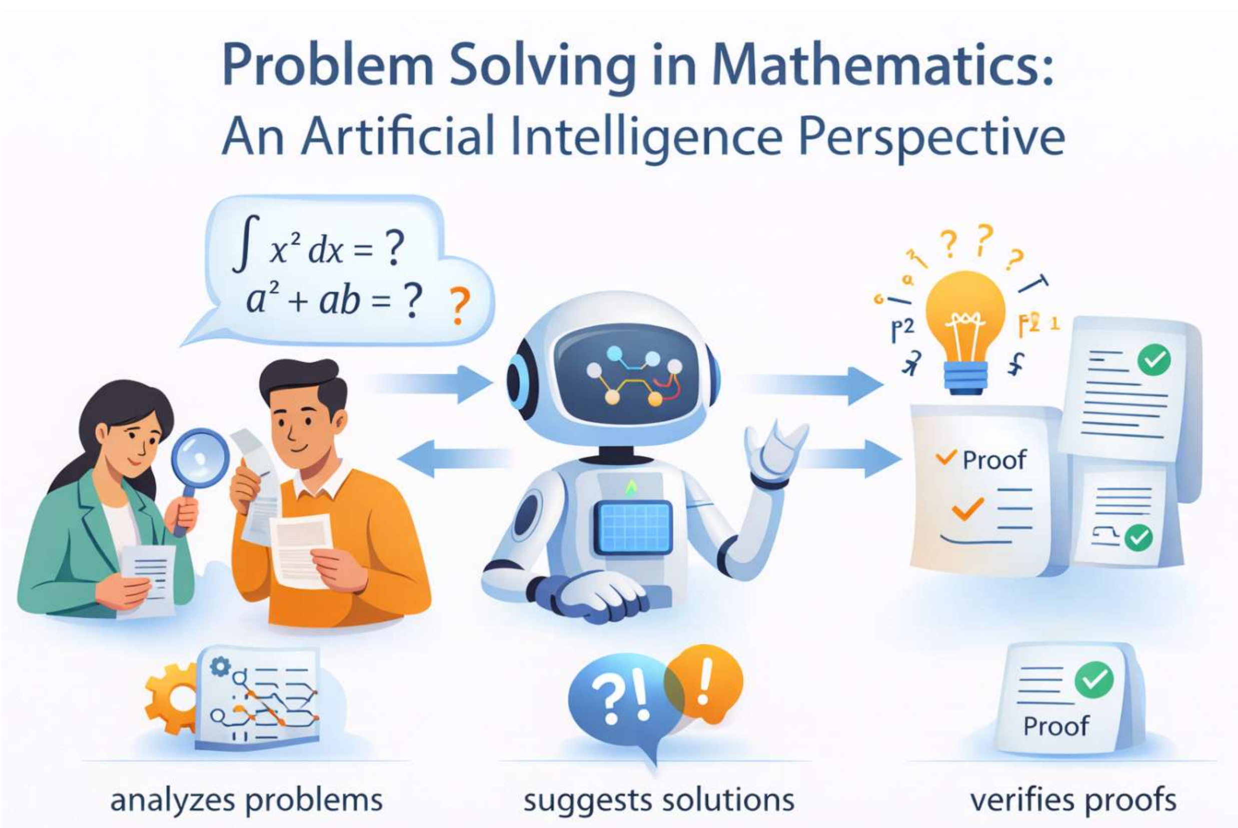

Figure1

. AI does not replace mathematical reasoning but acts as a powerful assistant that enhances creativity, efficiency, and accuracy. By combining logical reasoning with data-driven learning, AI opens new directions for mathematical discovery, education, and interdisciplinary research.

3.1.AI-Driven Mathematical Modeling and Automated Problem Solving

AI-driven mathematical modeling and automated problem solving signify one of the most revolutionary applications of artificial intelligence in the field of mathematics. Traditionally, mathematical modeling depended on manually derived equations, domain expertise, and custom numerical solvers. Although effective, these methods frequently encounter difficulties with highly nonlinear, high-dimensional, or data-sparse systems. Recent AI techniques tackle these challenges by learning models directly from data while still integrating mathematical structure and constraints, which facilitates quicker, more adaptable, and more scalable solutions [1]. In the realm of mathematical modeling, AI systems are increasingly employed to deduce governing equations, parameters, or solution operators from observed data. Machine learning models, especially deep neural networks, can approximate intricate functional relationships that are challenging to articulate analytically. When paired with mathematical constraints such as conservation laws or boundary conditions, these models transform into dependable tools for scientific and engineering applications. This transition enables mathematicians to shift from exclusively equation-driven modeling to hybrid methodologies where data and theory collaboratively shape the model structure [2]. Automated problem solving represents another significant advancement made possible by AI. Symbolic AI systems are now capable of assisting in algebraic manipulation, equation solving, and even theorem proving. Neural-symbolic approaches merge the pattern recognition abilities of neural networks with rule-based symbolic reasoning, allowing systems to resolve mathematical problems incrementally rather than yielding opaque numerical outputs [3]. Such systems can generate solution strategies, verify intermediate steps, and adapt to new categories of problems, which is especially beneficial in research and education. AI has also greatly enhanced numerical problem solving. Neural networks are being utilized to approximate solutions to ordinary and partial differential equations, often achieving faster results than traditional methods.

. AI has also significantly improved numerical problem solving. Neural networks are being used to approximate solutions to ordinary and partial differential equations, often achieving faster inference than classical solvers once trained. These methods are especially effective in problems requiring repeated simulations, such as optimization, control, or uncertainty analysis. By learning solution operators rather than single solutions, AI models can generalize across parameter spaces and initial conditions [4].

Another important dimension is automation and scalability. AI-based mathematical solvers can operate in real time, adapt to streaming data, and scale to large systems where traditional methods become computationally prohibitive. This capability is critical in areas such as climate modeling, financial mathematics, and biological systems, where models must evolve as new data becomes available [5]. Figure1.1 given below. Figure 1.1 illustrates the conceptual relationship between **symbolic computation** and its core contributing areas **mathematical software, computer algebra,** and **computational logic**. These three domains overlap to form the foundation of symbolic computation, highlighting how algebraic methods, logical reasoning, and software systems work together to manipulate mathematical symbols rather than numerical approximations. The cloud-shaped region represents the broader research area, emphasizing its interdisciplinary nature.

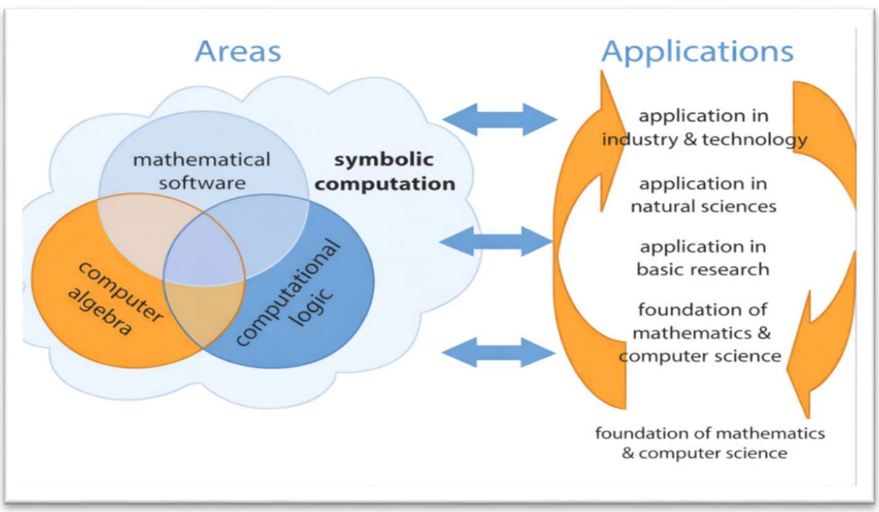

Figure 1.1 **AI-Driven Mathematical Modeling and Automated Problem Solving**

On the right side, the illustration depicts the uses of symbolic computation, linked by bidirectional arrows. This signifies a reciprocal relationship: progress in symbolic computation fuels applications, whereas practical challenges inspire additional theoretical and computational advancements. The applications encompass industry and technology, natural sciences, fundamental research, as well as the underpinnings of mathematics and computer science, showcasing the extensive significance of symbolic methods in both applied and theoretical fields.

3.2 .Machine Learning for Mathematical Pattern Discovery and Conjecture Generation

Machine learning has emerged as a powerful tool for discovering hidden patterns in mathematical data and supporting the generation of new conjectures. Traditionally, mathematical pattern discovery relied heavily on human intuition, manual exploration of examples, and deep theoretical insight. While these methods have driven centuries of mathematical progress, they are time-consuming and often limited by cognitive constraints. Machine learning introduces a complementary, data-driven paradigm that can systematically analyze large mathematical datasets, identify regularities, and propose hypotheses that may not be immediately apparent to human researchers [1]. At the core of machine learning–based pattern discovery is the ability to process vast collections of mathematical objects such as numerical sequences, algebraic expressions, graphs, and geometric configurations. Supervised and unsupervised learning models can extract relationships among variables, classify structures, and cluster objects based on intrinsic similarities. For example, neural networks and decision-tree–based models have been applied to integer sequences, polynomial families, and combinatorial objects to uncover recurring patterns and latent rules. These learned patterns often serve as the foundation for formulating conjectures about general properties or asymptotic behavior [2]. Conjecture generation represents a significant conceptual shift in mathematical research. Instead of starting from a theoretical framework and testing implications, machine learning systems begin with data and infer plausible general statements. In this context, symbolic regression plays an important role. By combining optimization with symbolic expression search, machine learning models can generate closed-form formulas that fit observed data. These formulas often resemble known mathematical laws or suggest entirely new relationships. Importantly, such conjectures are not proofs but hypotheses that guide mathematicians toward promising directions for formal verification [3]. Another influential approach involves neural-symbolic systems, which integrate statistical learning with symbolic reasoning. Neural components excel at pattern recognition and generalization, while symbolic components enforce logical consistency and mathematical syntax. This hybrid design allows AI systems to propose conjectures that are structurally meaningful rather than arbitrary numerical fits. For example, models can learn invariants, monotonicity properties, or algebraic identities that hold across multiple instances. These capabilities align closely with how mathematicians explore examples to gain intuition before proving results [4]. Machine learning has also been applied to graph- and geometry-based mathematical domains, where patterns are often highly complex. In topology, geometry, and combinatorics, data representations may involve high-dimensional or non-Euclidean structures. Graph neural networks and geometry-aware models can detect symmetries, equivalence classes, and recurring motifs that suggest deeper mathematical principles. Such insights are particularly valuable in areas like knot theory, network theory, and discrete geometry, where visual or combinatorial complexity makes manual exploration difficult [5].

Beyond pure mathematics, conjecture generation using machine learning has practical implications for scientific discovery. In mathematical physics, materials science, and dynamical systems, AI-driven pattern discovery has been used to infer conservation laws, scaling relations, and empirical formulas from simulation or experimental data. These conjectured relations often inspire new theoretical models or refine existing ones. By accelerating the cycle of hypothesis generation and testing, machine learning acts as a catalyst for interdisciplinary research [6]. Despite its promise, machine learning–based conjecture generation faces several challenges. One major concern is interpretability: patterns discovered by black-box models may be difficult to translate into mathematically meaningful statements. There is also the risk of overfitting, where models identify spurious correlations that do not generalize beyond the observed data. Addressing these issues requires careful dataset design, cross-validation, and integration of mathematical constraints into learning objectives. Additionally, human oversight remains essential, as conjectures must ultimately be evaluated, refined, and proven using rigorous mathematical reasoning [7]. Looking ahead, the role of machine learning in mathematical pattern discovery is expected to expand significantly. Future research aims to develop models that not only propose conjectures but also assess their plausibility, suggest proof strategies, and identify minimal counterexamples. The integration of machine learning with automated theorem proving and formal verification systems may further close the gap between conjecture and proof. In this evolving landscape, machine learning does not replace mathematicians but augments their creativity, enabling exploration at a scale and speed previously unattainable [8]. The figure2 demonstrates how mathematical expressions can be transformed into tree-structured representations, where each operation forms an internal node and each constant or variable appears as a leaf node. Arithmetic operations such as addition and multiplication, as well as functions like powers and trigonometric terms, are organized hierarchically to reflect the compositional structure of mathematics. This representation preserves the logical relationships between components of an expression, allowing complex formulas to be broken down into simpler sub expressions. By encoding mathematics in tree form, neural networks can process symbolic expressions rather than relying solely on numerical inputs. As a result, AI systems gain the ability to analyze, manipulate, and reason about mathematical structures in a way that resembles human mathematical intuition.

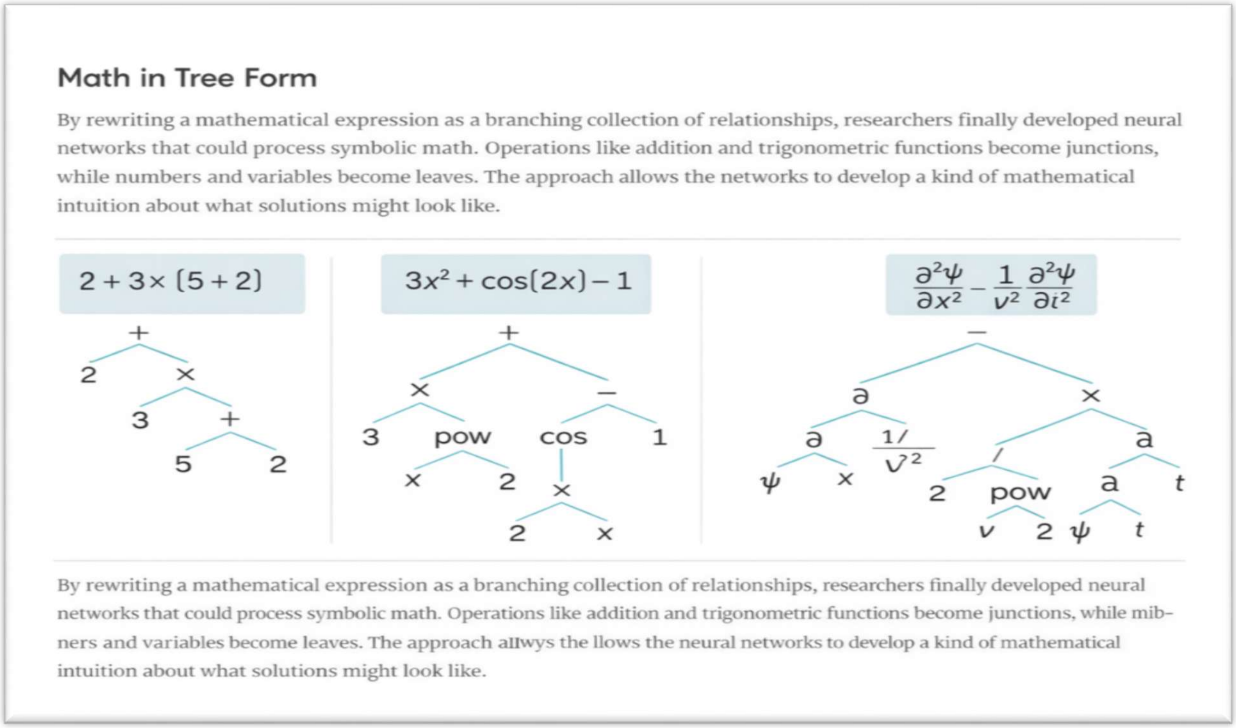

Figure :2 Math Expressions as branching diagrams

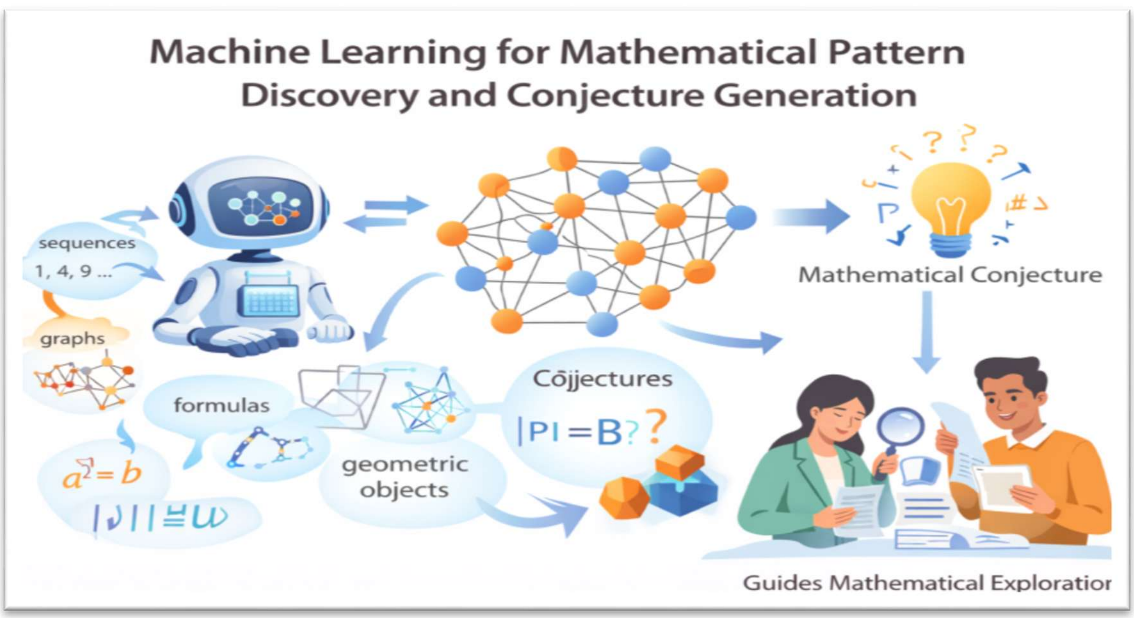

Figure : 2.1 ML for Mathematical Pattern

3.3. Deep Learning Approaches for Solving Differential and Integral Equations

Differential and integral equations form the mathematical foundation for modeling physical, biological, and engineering systems. Classical numerical techniques such as finite difference, finite element, and spectral methods have been widely used to solve these equations. However, these traditional approaches often face challenges when dealing with high-dimensional problems, complex geometries, or repeated simulations across varying parameters. In recent years, deep learning has emerged as a powerful alternative, offering flexible and data-driven methods for solving differential and integral equations efficiently [1]. Fig (3) One of the most influential deep learning frameworks in this area is the physics-informed neural network (PINN). PINNs embed the governing differential equations, boundary conditions, and initial conditions directly into the loss function of a neural network. Instead of learning solely from labeled data, the network is trained to minimize the residuals of the differential equations, ensuring physical consistency of the solution. In Fig 3 this approach reduces the need for large datasets and allows models to generalize across continuous domains. PINNs have been successfully applied to ordinary differential equations, partial differential equations, and integral equations in fluid dynamics, heat transfer, and electromagnetics [2].Another major advancement is operator learning, which shifts the focus from learning individual solutions to learning mappings between function spaces. Neural operator frameworks enable deep networks to approximate solution operators of differential equations rather than discrete solutions at fixed points. By learning these operators, models can rapidly predict solutions for new initial or boundary conditions without retraining. This paradigm significantly accelerates computation in scenarios that require repeated evaluations, such as optimization and uncertainty quantification [3].Deep learning methods also address the curse of dimensionality that limits classical solvers. High-dimensional differential and integral equations commonly arise in stochastic processes, financial mathematics, and kinetic theory. Neural networks can approximate high-dimensional functions using relatively compact representations, making them suitable for problems that are computationally intractable for grid-based methods. Additionally, deep architectures can adapt to irregular domains and complex boundary conditions, further extending their applicability [4]. Integral equations benefit from deep learning through neural approximations of integral operators and kernel functions. By learning these operators directly from data or simulations, AI-based solvers can handle nonlocal interactions efficiently.

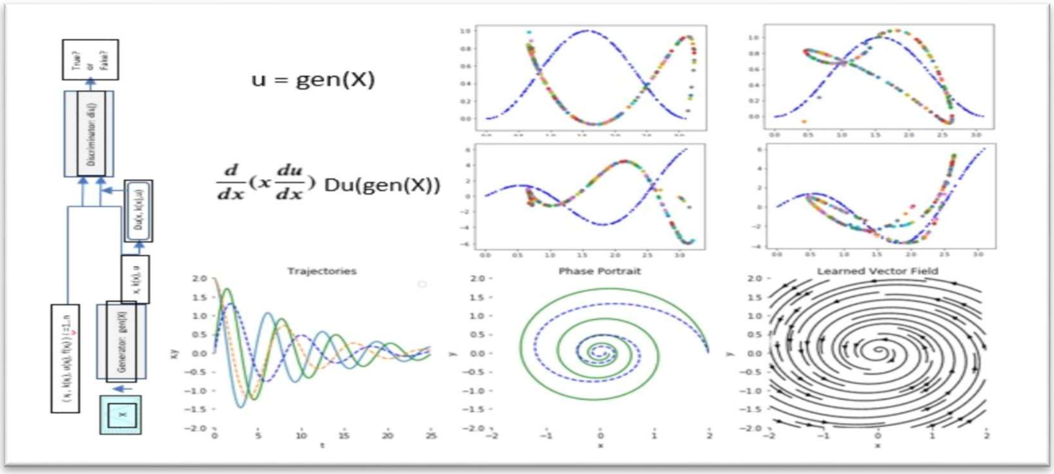

Figure:3 Deep Learning Approaches for Solving Differential and Integral Equations

Such approaches are particularly useful in materials science and biological systems, where interactions often span multiple spatial or temporal scales. Moreover, combining neural solvers with classical numerical methods leads to hybrid schemes that balance accuracy, interpretability, and computational efficiency [5].Despite their promise, deep learning approaches for solving differential and integral equations face several open challenges. Training stability, convergence guarantees, and error estimation remain active areas of research. Furthermore, integrating domain knowledge effectively and ensuring generalization beyond training regimes are critical for reliable deployment. Ongoing research aims to establish stronger theoretical foundations, develop adaptive architectures, and create benchmarks for systematic evaluation [6].

3.4. Geometry and Topology-Based Representation Learning in Mathematics

Geometry- and topology-based representation learning focuses on using the intrinsic shape, structure, and connectivity of mathematical objects to build meaningful data representations. Instead of relying only on numerical features, these approaches model data as geometric entities such as graphs, manifolds, or surfaces, and analyze their topological properties like holes, connected components, and orientability. As illustrated by classical surfaces such as the sphere, torus, double torus, cross-cap, and Klein bottle, topology distinguishes objects based on invariant structural characteristics rather than precise measurements. This perspective is especially powerful in mathematics, where many problems depend on structure that remains unchanged under continuous deformation.In modern learning systems, geometric representations allow models to respect symmetries and spatial relationships, while topological features capture global patterns that are insensitive to noise or distortion. Together, they enable more robust, interpretable, and mathematically grounded learning frameworks. These methods are increasingly used to study complex mathematical datasets, discover patterns, and support conjecture formation by identifying stable structural regularities.The future applications of geometry- and topology-based representation learning are wide-ranging. In mathematics, they can assist in automated theorem discovery, classification of high-dimensional geometric objects, and analysis of algebraic and topological invariants. In scientific computing, they support physically consistent simulations and shape-aware modeling. Advantages of these approaches include strong invariance properties, reduced dependence on large training datasets, improved generalization, and enhanced interpretability of learned representations. As theoretical understanding and computational tools continue to advance, geometry- and topology-driven learning is expected to play a key role in bridging abstract mathematical theory with data-driven artificial intelligence systems. figure 4 illustrates a set of representative **topological surfaces** commonly used to explain fundamental concepts in topology. The **sphere** represents a simply connected surface with no holes, serving as a basic example of a closed and orientable manifold. The **torus** introduces a single hole, demonstrating how topology classifies shapes based on connectivity rather than precise geometry. The **double torus** extends this idea by containing two holes, highlighting the notion of *genus,* which counts the number of holes in a surface. The **cross-cap surface** depicts a non-orientable surface, meaning it lacks a consistent notion of "inside" and "outside," unlike the sphere or torus. The **Klein bottle** is another non-orientable surface that cannot be embedded in three-dimensional space without self-intersection, emphasizing abstract topological properties over physical realizability. Together, these surfaces show how topology groups shapes according to invariant properties such as holes and orientability, rather than size or curvature. In the context of topological data analysis and geometry-aware learning, such examples motivate the use of topological invariants to characterize complex data structures in a deformation-invariant and interpretable manner.

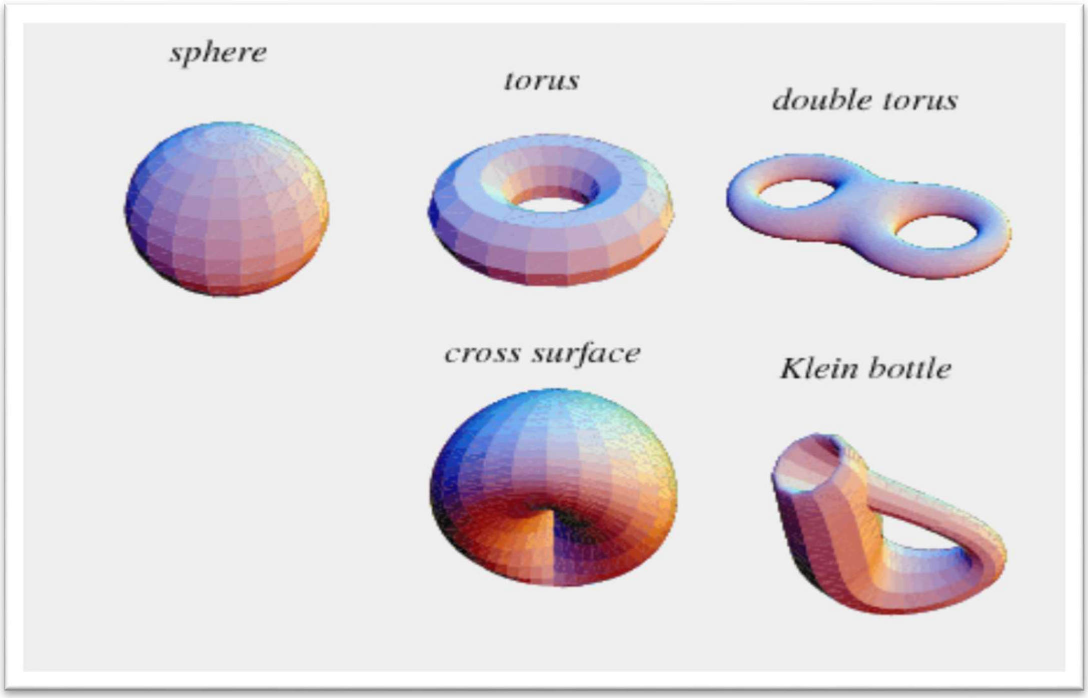

Figure:4 Geometry and Topology-Based Representation

3.5 Optimization and Dynamical Systems Guided by Artificial Intelligence

Optimization and dynamical systems form the mathematical backbone of many artificial intelligence algorithms, particularly in deep learning. Training a neural network can be viewed as solving a high-dimensional optimization problem, where the objective is to minimize a loss function over a complex, nonconvex landscape. Classical optimization techniques often struggle in such settings due to local minima, saddle points, and unstable convergence behavior. Artificial intelligence has introduced new perspectives and tools that guide optimization processes using data-driven insights and dynamical systems theory [1]. From a dynamical systems viewpoint, learning algorithms such as gradient descent can be interpreted as discrete-time approximations of continuous-time differential equations. This interpretation allows researchers to analyze stability, convergence, and long-term behavior using tools from dynamical systems and control theory. By studying optimization trajectories as flows in parameter space, AI-guided methods can identify stable attractors, avoid unstable regions, and design learning rules that converge more reliably. Such mathematically informed analysis helps explain why certain optimization algorithms generalize better despite operating in highly nonconvex regimes [2].

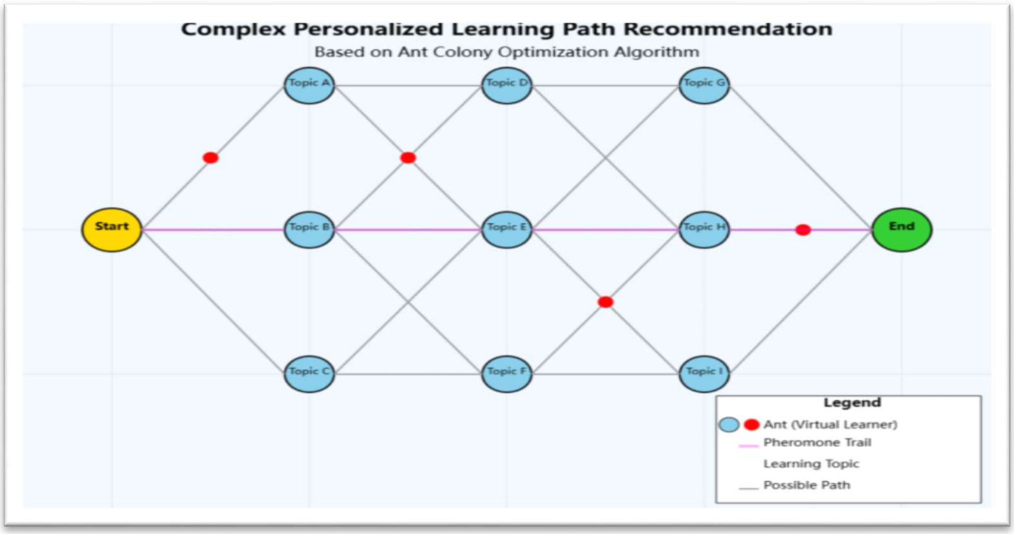

Figure:5 Optimization and dynamical systems

Artificial intelligence also contributes to the design of adaptive and intelligent optimization strategies. Instead of relying on fixed update rules, AI-based optimizers learn how to optimize by adapting step sizes, momentum terms, and update directions based on past experience. These learned optimizers can adjust to different problem structures and scales, leading to faster convergence and improved robustness. This approach is particularly useful in large-scale models and real-time applications where computational efficiency is critical [3].In addition, AI-guided optimization plays an important role in handling constrained and multi-objective problems. Many real-world systems involve physical, economic, or safety constraints that must be respected during optimization. By integrating constraints directly into the learning process, AI-driven optimization frameworks can ensure feasibility while still achieving high performance. Dynamical systems theory provides a principled way to enforce stability and constraint satisfaction, especially in control and reinforcement learning applications [4].The interaction between optimization and dynamical systems is also central to understanding generalization and implicit regularization in deep learning. Research shows that the dynamics of training algorithms bias solutions toward simpler or more structured representations, even without explicit regularization terms. This implicit bias can be analyzed using mathematical tools such as Lyapunov functions and stability analysis, offering deeper insight into why AI models perform well on unseen data [5]. Despite these advances, several challenges remain. Theoretical guarantees for convergence and stability are still limited for many AI-guided optimization methods, particularly in nonconvex and stochastic settings. Furthermore, bridging the gap between continuous-time theory and discrete-time algorithms requires careful mathematical treatment. Ongoing research aims to unify optimization theory, dynamical systems, and machine learning into a coherent framework that supports scalable, interpretable, and reliable AI systems

4.Discussion

The five topics discussed collectively illustrate how artificial intelligence is reshaping mathematics by introducing new methodologies that complement and extend classical mathematical approaches. Together, they highlight a clear transition from manually driven, equation-centric methods toward data-assisted, structure-aware, and adaptive mathematical reasoning. AI-driven mathematical modeling and automated problem solving demonstrate how learning-based systems can overcome limitations of traditional modeling techniques, especially in nonlinear, high-dimensional, or data-scarce scenarios. By learning governing equations, parameters, or solution operators directly from data while embedding mathematical constraints, AI enables hybrid modeling frameworks that combine theory with empirical evidence. This shift significantly enhances scalability, adaptability, and computational efficiency, particularly in domains such as climate science, finance, and biological systems. Automated problem solving further reduces human effort in routine symbolic manipulation and equation solving, allowing mathematicians to focus on higher-level reasoning and theory development. Machine learning for mathematical pattern discovery and conjecture generation introduces a paradigm where data exploration systematically supports mathematical creativity. By analyzing large collections of sequences, expressions, graphs, and geometric objects, machine learning models uncover hidden regularities that may elude manual inspection. These discovered patterns form the basis for conjectures, guiding mathematicians toward promising hypotheses. Neural-symbolic systems and symbolic regression are especially important in ensuring that generated conjectures remain mathematically meaningful rather than purely numerical artifacts. This synergy between human intuition and machine-driven exploration accelerates discovery while preserving mathematical rigor. Deep learning approaches for solving differential and integral equations represent a major advancement in applied mathematics and scientific computing. Methods such as physics-informed neural networks and neural operators provide flexible alternatives to classical numerical solvers, particularly for high-dimensional or parameterized problems. By learning solution operators instead of single solutions, these approaches enable rapid inference across varying conditions, which is critical for optimization, control, and uncertainty analysis. Although challenges remain in stability and theoretical guarantees, these methods significantly expand the scope of solvable mathematical models.Geometry and topology-based representation learning emphasizes the importance of structure in mathematical data. By leveraging geometric properties such as symmetry and equivariance, along with topological invariants that capture global features, AI systems can learn robust and interpretable representations of complex mathematical objects. This is particularly valuable in graph-based, manifold-based, and combinatorial domains, where traditional vector representations fail to capture intrinsic relationships. The integration of geometry and topology strengthens generalization and reduces data requirements by embedding mathematical priors into learning architectures.

5.Future Scope

The future of artificial intelligence in mathematics is poised for significant expansion as learning-based methods become more deeply integrated with rigorous mathematical theory. One of the most promising directions is the development of hybrid frameworks that seamlessly combine data-driven learning with symbolic reasoning and analytical models. Such systems will enable AI to not only approximate solutions but also reason about mathematical structures, generate explanations, and support formal verification. This integration is expected to enhance reliability, interpretability, and trustworthiness in mathematical AI applications. Another important future direction lies in scalable and theoretically grounded learning methods. While current AI-based solvers demonstrate impressive empirical performance, stronger mathematical guarantees for stability, convergence, and generalization remain essential. Advancing optimization theory, dynamical systems analysis, and learning theory will help establish these guarantees, particularly in high-dimensional and nonconvex settings. Additionally, the development of differentiable and efficient topological and geometric layers will further strengthen structure-aware representation learning.AI-assisted conjecture generation and automated theorem proving are also expected to mature significantly. Future systems may autonomously propose conjectures, evaluate their plausibility, suggest proof strategies, and identify counterexamples, thereby accelerating mathematical discovery. In applied mathematics and scientific computing, AI-driven solvers for differential and integral equations will continue to evolve, enabling real-time simulations and data-adaptive modeling in complex systems.

6.Conclusion

The integration of artificial intelligence into mathematics marks a profound transformation in the way mathematical problems are modeled, analyzed, and solved. This study has explored key emerging areas, including AI-driven mathematical modeling, automated problem solving, machine learning–based pattern discovery, deep learning approaches for differential and integral equations, geometry and topology-based representation learning, and optimization guided by dynamical systems. Together, these themes demonstrate that AI serves not as a replacement for mathematical theory, but as a powerful extension that enhances mathematical reasoning and computational capability.AI-driven modeling frameworks enable mathematicians to address highly nonlinear, high-dimensional, and data-sparse problems that challenge traditional approaches. Automated symbolic reasoning and neural–symbolic systems support algebraic manipulation and conjecture generation, accelerating discovery while maintaining mathematical structure. Deep learning methods for differential and integral equations provide efficient and flexible alternatives to classical numerical solvers, particularly for large-scale and parameterized problems. Meanwhile, geometry and topology-based learning introduce structure-aware representations that capture invariance, symmetry, and global properties, leading to improved robustness and interpretability. Optimization and dynamical systems perspectives further strengthen AI methodologies by offering insights into training stability, convergence, and implicit regularization. These mathematical lenses help explain the success of modern learning algorithms and guide the design of more reliable and efficient optimization strategies. Despite these advances, challenges remain in establishing theoretical guarantees, scalability, and interpretability, underscoring the need for continued interdisciplinary research.

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