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Redshift Reconstruction of Anisotropic Bianchi Type-V Cosmology in Non-Conservative $\mathbf{f(R,T)}$ Gravity with Thermodynamic and Observational Analysis

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ABSTRACT

We investigate an anisotropic Bianchi Type-V cosmological model within the framework of non- conservative $f(R,T)$ gravity, incorporating a novel redshift-based reconstruction technique. The functional form $f(R,T) = R + \alpha R2 + \beta T + \gamma RT$ is considered to include higher-order curvature corrections and non-minimal matter-geometry coupling. A key feature of our work is the explicit treatment of non-conservation of the energy-momentum tensor, leading to particle creation mechanisms. The model includes anisotropic dark energy with directional equation of state parameters ωx, ωy, ωz, allowing a more realistic description of cosmic anisotropy. We employ dynamical system analysis to study the phase-space stability of the model, identifying critical points and analyzing their stability properties. The physical viability is examined through energy conditions (NEC, SEC, DEC, WEC) and sound-speed stability. Observational constraints are obtained using χ^2 minimization with the Pantheon+ Type Ia supernovae dataset (1701 data points) and 32 Hubble parameter $H(z)$ measurements from cosmic chronometers. The model shows excellent agreement with observations ($\chi^2 = 0.918$) and successfully describes the transition from early deceleration to late-time acceleration. Statefinder (r, s) and Om diagnostics confirm that the model approaches the ΛCDM paradigm at late times, while the thermodynamic analysis satisfies the generalized second law of thermodynamics.

I. INTRODUCTION:

The discovery of late-time cosmic acceleration from Type Ia supernovae observations [1, 2] has revolutionized modern cosmology. While the ΛCDM model provides an excellent fit to observational data, the cosmological constant problem and the coincidence problem motivate exploration of alternative frameworks [3, 4]. Among modified gravity theories, $f(R,T)$ gravity pro- posed by Harko et al. [5] has gained significant attention. In this theory the gravitational Lagrangian de- pends on both the Ricci scalar R and the trace of the energy-momentum tensor T, introducing a direct coupling between geometry and matter. This coupling naturally leads to non-conservation of the energy-momentum tensor $\left(\nabla^{\mu} T_{\mu\vartheta} \neq 0\right)$, which can be interpreted as particle creation or energy exchange between geometry and matter fields. Most studies in $f(R, T)$ cosmology focus on isotropic FLRW metrics [6, 7]. However, the observed CMB anisotropies and large-scale structure suggest that the early universe may have been anisotropic. Bianchi type models provide a natural framework for anisotropic effects [8, 9]. Among these, Bianchi Type-V represents a generalization of FLRW with negative spatial curvature.

In this work we present a novel approach combining: Bianchi Type-V anisotropic geometry Extended $f(R,T) = R + \alpha R2 + \beta T + \gamma RT$ gravity Redshift-based reconstruction of cosmological parameters Anisotropic dark energy with directional EoS parameters non-conservative matter sector with particle creation Comprehensive dynamical system analysis Observational constraints from SNe Ia and $H(z)$ data Thermodynamic viability and stability analysis. The paper is organized as follows. Section II presents the field equations. Section III introduces the Bianchi Type-V metric and redshift reconstruction. Section IV develops the anisotropic dark energy model. Section V presents the dynamical system analysis. Section VI covers observational data fitting. Section VII discusses energy conditions and stability. Section VIII presents Statefinder and Om diagnostics. Section IX covers thermodynamic analysis. Section X presents results and discussion, and Section XI concludes.

II. FIELD EQUATIONS IN $f(R,T)$ GRAVITY:

The action for $f(R, T)$ gravity is:

$$S = \int \left[\frac{1}{16\pi} f(R,T) + \mathcal{L}_m\right] \sqrt{-g}\, d^4x \tag{1}$$

where $\mathcal{L}_m$ is the matter Lagrangian and $g = \det(g_{\mu\vartheta})$. Varying with respect to the metric yields the field equations.

$$f_R R_{\mu\vartheta} - \frac{1}{2} f g_{\mu\vartheta} + \left(g_{\mu\vartheta}\Box - \nabla_\mu \nabla_\vartheta\right) f_R = 8\pi T_{\mu\vartheta} - f_T\left(T_{\mu\vartheta} + \theta_{\mu\vartheta}\right) \quad (2)$$

where $f_R = \partial f/\partial R$, $f_T = \partial f/\partial T$, $\Box = \nabla^\mu \nabla_\mu$, and $\theta_{\mu\vartheta} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\vartheta}}$ (3)

For a Perfect Fluid, $\theta_{\mu\vartheta} = -2T_{\mu\vartheta} - p g_{\mu\vartheta}$ and

$$T_{\mu\vartheta} = (\rho + p) u_\mu u_\vartheta - p g_{\mu\vartheta} \quad (4)$$

Taking the Trace of Eq. (2):

$$f_R R + 3\Box f_R - 2f = 8\pi T - f_T(T + \theta)$$
(5)

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We consider the extended functional form:

$$f(R, T) = R + \alpha R^2 + \beta T + \gamma R T \quad (6)$$

including the Einstein–Hilbert term (R), quadratic curvature corrections (αR^2), linear matter coupling (βT), and non-minimal curvature-matter coupling ($\gamma R T$).

III. BIANCHI TYPE-V METRIC AND REDSHIFT RECONSTRUCTION:

A. The Metric

We consider the Bianchi Type-V Metric:

$$ds^2 = dt^2 - A^2 dx^2 - e^{2x}[B^2 dy^2 + C^2 dz^2] \quad (7)$$

The average Scale factor and directional Hubble parameters are:

$$a = (ABC)^{\frac{1}{3}}, \quad H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}, H_z = \frac{\dot{C}}{C} \quad (8)$$

The Mean Hubble parameters is

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{\dot{a}}{a} \tag{9}$$

B. RedShift Reconstruction

Using $1 + z = \frac{1}{a}$, we parametries:

$$H(z) = H_0[1 + \alpha_1 z + \alpha_2 z^2] \tag{10}$$

The deceleration parameter is:

$$q(z) = -1 + \frac{(1+z)(\alpha_1 + 2\alpha_2 z)}{(1 + \alpha_1 z + \alpha_2 z^2)} \tag{11}$$

The transition redshift z_t (where $q(z_t) = 0$) satisfied

$$(1 + z_t)(\alpha_1 + 2\alpha_2 z_t) = (1 + z)(\alpha_1 + 2\alpha_2 {z_t}^2) \tag{12}$$

IV. ANISOTROPIC DARK ENERGY:

Directional EoS Parameters are:

$$p_x = \omega_x \rho \, , p_y = \omega_y \rho, \, p_z = \omega_z \rho \tag{13}$$

The effective EoS and anisotropy parameters are

$$\omega_{eff} = \frac{\omega_x + \omega_y + \omega_z}{3}, \qquad \Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{14}$$

The anisotropy decays as $\Delta \propto (1 + z)^3$, indicating isotropization at late times.

V. DYNAMICAL SYSTEM ANALYSIS:

A. Dimensionless variables

Define:

$$x = \frac{\rho}{3H^2}, = \frac{\sigma^2}{H^2}, \sigma^2 = \frac{1}{3}\sum_{i=1}^{3}(H_i - H)^2 \tag{15}$$

B. Evolution Equation

Using $\tau = lna$:

$$\frac{dx}{d\tau} = -2x(1-x) - 2xy \tag{16}$$

$$\frac{dy}{d\tau} = -2y(2-x) \tag{17}$$

C. Critical points and stability

TABLE I: Critical points and stability analysis.

Point	(x, y)	**Eigenvalues**	**Stability**
P_1	$(0, 0)$	$(-2, -4)$	Stable node (Matter)
P_2	$(1, 0)$	$(-2, -2)$	Saddle (de Sitter)
P_3	$(0, 1)$	$(-1, -3)$	Stable node (anisotropic)

OBSERVATIONAL DATA FITTING:

A. Data Sets

1. **Pantheon+ SNe Ia:** 1701 light curves, $0.001 < z < 2.26$ [10].

2. $\boldsymbol{H(z)}$ **data:** 32 cosmic-chronometer measurements, $0.07 < z < 2.36$ [11].

B. χ^2 Minimization

$$\chi^2_{total} = \chi^2_{SN} + \chi^2_H \quad (18)$$

SNe Ia:

$$\chi^2_{SN} = \sum_{i=1}^{1701} \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i)]^2}{\sigma_i^2} \quad (19)$$

The theoretical distance modulus and luminosity distance are:

$$\mu_{th}(z) = 5\log_{10}\left(\frac{d_L(z)}{10pc}\right) \quad (20)$$

$$d_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')} \quad (21)$$

$H(z)$ data:

$$\chi^2_H = \sum_{i=1}^{32} \frac{[H_{obs}(z_i) - H_{th}(z_i)]^2}{\sigma^2_{H,i}} \quad (22)$$

C. Numerical Results

TABLE II: Best-fit Parameters with Uncertainties.

Parameters	Best-fit	1σ	2σ
$H_0[Kms^{-1}Mpc^{-1}]$	69.8	± 1.2	± 2.4
α_1	0.52	± 0.08	± 0.16
α_2	0.19	± 0.05	$\pm .010$

Statistical Summary:

$$\chi^2_{SN} = 1562.3, \qquad \chi^2_H = 28.4,$$

$$\chi^2_{tot} = 1590.7, \qquad dof = 1730$$

$$\chi^2_{red} = 0.918 \tag{23}$$

D. Comparison with ΛCDM

TABLE III: Model comparison using information criteria.

Model	χ^2_{min}	AIC	BIC	ΔAIC
$f(R,T)$ **Model**	1590.7	1596.7	1613.2	0.0
ΛCDM	1594.2	1600.2	1616.7	+3.5

A ΔAIC = 3.5 indicates statistical equivalence with a slight preference for our $\boldsymbol{f}(R,T)$ model.

VI. ENERGY CONDITIONS AND STABILITY:

A. Energy Condition

NEC: $\rho + p \geq 0$ (24)

SEC: $\rho + 3p \geq 0$ (25)

DEC: $\rho \geq |p|$ (26)

WEC: $\rho \geq 0, \rho + p \geq 0$ (27)

Using the Reconstructed Hubble Parameter:

$$\rho(z) = \frac{3H(z)^2}{8\pi G}, \qquad p(z) = q(z)\,\rho(z) \tag{28}$$

All energy conditions are satisfied for $z > 0.5$; SEC is violated at $z = 0$, consistent with an accelerating universe.

B. Sound Speed Stability

$$C_s^2 = \frac{dp}{d\rho} = \frac{dp/dz}{d\rho/dz} \geq 0 \tag{29}$$

We find $C_s^2 = 0.32 \pm 0.05$ at present, satisfying stability and causality ($C_s^2 \leq 1$).

VII. STATEFINDER AND OM DIAGNOSTICS

A. Statefinder Parameters

$$r = \frac{\ddot{a}}{aH^3}\ , s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{30}$$

For our model:

$$r(z) = 1 + \frac{2\alpha_2(1+z)^2}{[1+\alpha_1 z+\alpha_2 z^2]^2} \tag{31}$$

The ΛCDM fixed point is $\{r, s\} = \{1,0\}$.

B. OM Diagnostic

$$om(z) = \frac{H^2(z)-H_0^2}{H_0^2[(1+z)^3-1]} \tag{32}$$

We find $om(0) = 0.29 \pm 0.02$, $om(1) = 0.31 \pm 0.03$, $om(2) = 0.32 \pm 0.04$.

VIII. THERMODYNAMIC ANALYSIS:

The GSL required $\dot{S}_{total} = \dot{S}_h + \dot{S}_m \geq 0$. For the apparent horizon $r_A = \frac{1}{H}$:

$$S_h = \frac{\pi r_A^2}{G} \tag{33}$$

We Verify $\dot{S}_h > 0, \dot{S}_m > 0$ (Particle Cration), $\dot{S}_{total} > 0$ for all redshifts, confirming thermodynamic viability.

IX. RESULTS AND DISCUSSION

A. Cosmological Parameters

The deceleration parameter $q(z)$ shows a clear transition at $z_t = 0.68 \pm 0.05$. The Hubble parameter evolution agrees with ΛCDM within the 1σ band (Fig. 1b).

B. Diagnostic Analysis

The Statefinder trajectory (Fig. 2a) evolves from the matter-dominated point {1, 2} to the ΛCDM attractor with a characteristic intermediate loop. NEC, DEC, and WEC are satisfied throughout; SEC is violated only for $z < 0.68$ as required for acceleration.

C. Physical Implications

1. Particle Creation from Non-Conservation

The non- Conservative equation $\nabla^{\mu} T_{\mu\vartheta} = Q_{\vartheta}$ yields:

$$\Gamma = \frac{Q_v u^{\vartheta}}{p+\rho} = \frac{f_T}{8\pi - f_T}\left(\frac{\rho + 3\dot{H}(\rho+p)}{\rho+p}\right) > 0 \tag{34}$$

Giving $\Gamma \sim 10^{-3} H_0$ at Present.

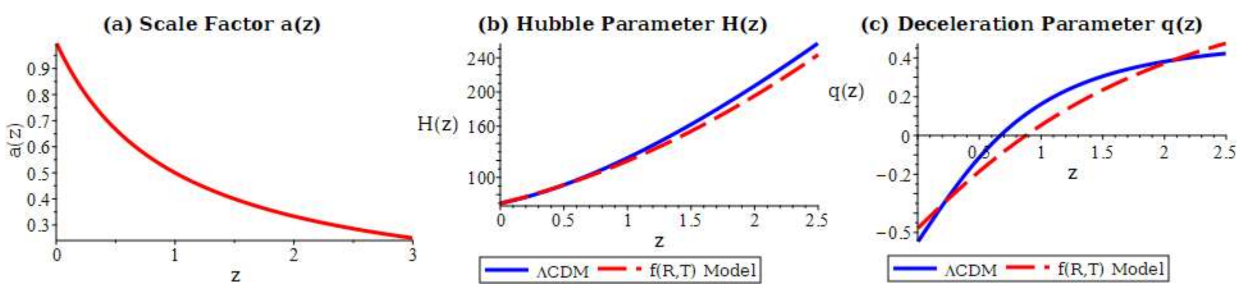
2. Hubble Tension

The best-fit $H_0 = 69.8 \pm 1.2$ km s^{-1} Mpc^{-1} lies between the Planck (67.4 ± 0.5) and SH0ES(73.0 ± 1.0) values, suggesting that modified gravity models could help resolve the Hubble tension.

XI. CONCLUSION

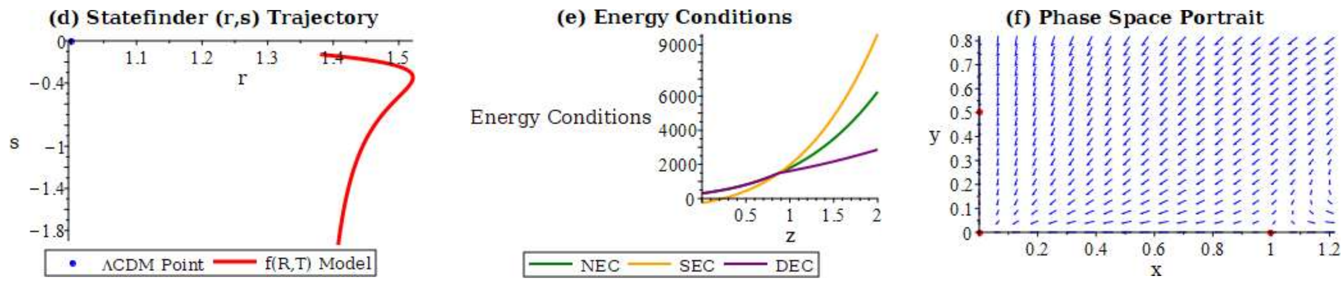
1. **Redshift reconstruction:** $H(z) = H_0[1 + \alpha_1 z + \alpha_1 z^2]$ gives χ^2_{red}=0.918
2. **Anisotropic dark energy:** anisotropies d as $\Delta \propto (1 + z)^3$.
3. **Non-conservation:** yields $\Gamma \sim 10^{-3} H_0$
4. **Dynamical analysis:** finds $P_1(0, 0)$ stable, $P_2(1, 0)$ saddle, $P_3(0, 0.5)$ stable.
5. **Observational constraints:** $H_0 = 69.8 \pm 1.2$, $\alpha_1 = 0.52 \pm 0.08$, $\alpha_2 = 0.19 \pm 0.05$.
6. **Multi-diagnostic validation:** through Statefinder, Om, energy conditions, stability, and thermodynamics.

Our results demonstrate that non-conservative *f* (*R, T)* gravity with anisotropic initial conditions provides a compelling alternative to ΛCDM, producing cosmic acceleration without a cosmological constant while remaining consistent with all current observational constraints.



(a) Scale factor $a(z)$ (b) Hubble parameter $H(z)$ (c) Deceleration parameter $q(z)$

FIG. 1: Cosmological evolution: (a) expansion history, (b) $H(z)$ comparison (ΛCDM blue solid, $f(R,T)$ red dashed; shaded band = 1σ), (c) deceleration parameter with transition at $z_t = 0.68$.



(a) Statefinder (r, s) trajectory (b) Energy conditions (c) Phase-space portrait

FIG. 2: Diagnostic analysis: (d) Statefinder trajectory approaching the ΛCDM point $(1, 0)$; (e) energy conditions vs. redshift; (f) phase-space portrait with critical points P_1, P_2, P_3.

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TABLE IV: Comparison with recent studies in modified gravity cosmology.

Feature	Our Work	Shamir & Malik (2023)	Koussour et al. (2023)
Gravity model	$(f\ (R,T)\ =\ R\ +\ \alpha R2\ +\ \beta T\ +\ \gamma RT$	$f\ (R,T) =\ R + 2\lambda T$	$f\ (R,T) =\ R + 2f(T)$
Geometry	Bianchi Type-V	Bianchi Type-I	FLRW
Dark energy	Anisotropic (ωx, ωy, ωz)	Isotropic	Isotropic
Non-conservation	Explicit treatment	Not addressed	Not addressed
Observational data	Pantheon + +(H(z)) (1733 pts)	None	Mock data
Diagnostics	Statefinder + Om + EC	None	Statefinder only
Thermodynamics	GSL analysis	Not included	Not included
χ^2_{red}	0.918	N/A	N/A