



Average Degree-Distance and Gutman Indices of Mycielskian Graph and its Complement

Vidya S. Raikar ^a and Manjula Patil ^b

^a Department of Mathematics, Government First Grade College and P G Studies Centre, Naragund, Karnataka

e-mail^a: vidya.gfgc@gmail.com

^bDepartment of Mathematics, Government First Grade College Dharwad, Karnataka

e-mail^c: manjulapatil2010@gmail.com

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ABSTRACT

For a graph $G = (V, E)$, the Mycielskian of G is the graph $\mu(G)$ (or simply μ) [6] is defined as the graph having vertex set $V \cup X \cup \{x\}$ and edge set $E \cup \{vixj : vivj \in E\} \cup \{xixj : 1 \leq j \leq n\}$, where $V = \{v_1, v_2, \dots, v_n\}$ and $X = \{x_1, x_2, \dots, x_n\}$. In this paper, we define a new distance based parameters viz., average degree-distance index $DDa(G)$ and average Gutman index $Gla(G)$ of molecular graph G and calculated $DDa(G)$ and $Gla(G)$ of Mycielskian graph and its compliment.

1 Introduction

All graphs-considered in this paper are finite, connected without loops or multiple edges. Let $G = (V, E)$ be graph of order n and size m . If $u, v \in V(G)$ are said to be adjacent if $e = uv \in E(G)$. The degree of a vertex $v \in V(G)$ is the number of edges incident to v in G . It is denote by $degG(v)$. Let $\bar{G}$ denote the complement of a graph G , where the order and size of $\bar{G}$ are n and m respectively. For $u, v \in V(G)$, the distance $d(u, v)$ is the length of the shortest path connectinsg the vertices u, v . If u and v belongs to different different components then the $d(u, v) = \infty$. The average distance of $u, v \in V(G)$ is defined as $\frac{1}{n(n-1)} \sum_{u \neq v} d(u, v)$. For undefined terminology in this paper we refer the interested reader to [5].



Graph theory has become one of the emerging field of Mathematics, due to its applications in different areas such as computer science, physics, chemistry, etc. One of the interesting application is in the field of Mathematical chemistry, in particular chemical graph theory. Due to the applications of topological indices in QSPR studies, this area

of research got immediate attention from the chemists as well as mathematician around the world.

The topological index is basically, some positive integer associated with the molecular graph. There are three types of topological indices viz.,

- Degree based topological indices
- Distance Based Topological Indices
- Matrix Based Topological Indices

In the year 1947, H. Wiener [8] has introduced the first topological index, unknowingly he used graph theory to predict the boiling point of certain paraffin alkanes.

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)$$

Later, the Gutman [4] conceived the Zagreb indices.

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v)$$

These parameters are the most studied topological indices in the field of Mathematical Chemistry. After, that many parameters have been put forward by the various researchers such as terminal Wiener index $TW(G)$ [3], degree-distance index $DD(G)$ [1], Gutman index $GI(G)$ [2], Ashwini index [7] etc., and now there are more than 1000 parameters have been introduced so far.

$$TW(G) = \sum_{\{u,v \subseteq V_T(G)\}} d(u,v \setminus G) = \sum_{1 \leq i < j \leq k} d(u,v \setminus G)$$

$$DD(G) = \sum_{\{u,v \subseteq V(G)\}} d_G(u,v)[deg_G(u) + deg_G(v)].$$

$$GI(G) = \sum_{\{u,v \subseteq V(G)\}} d_G(u,v)[deg_G(u)deg_G(v)].$$

$$\mathcal{A}(T) = \sum_{1 \leq i < j \leq n} d_T(v_i, v_j)[deg_T(N(u_i)) + deg_T(N(v_j))].$$

Where $N(v) = \{u \in V(G) : uv \in E(G)\}$.

In this connection, we are defining a new distance based topological index namely, the average degree-distance index of molecular graph as follows:

$$DD^a(G) = \frac{1}{n(n-1)} \sum_{\{u,v \subseteq V(G)\}} d(u,v)[deg(u) + deg(v)]$$

2 Mycielskian Graph

For a graph $\mathbf{G} = (V, E)$, the Mycielskian of $\mathbf{G}$ is the graph $\mu(\mathbf{G})$ (or simply μ) [6] is defined as the graph having vertex set $V \cup X \cup \{x\}$ and edge set $E \cup \{v_i x_j : v_i v_j \in E\} \cup \{x_i x_j : 1 \leq j \leq n\}$, where $V = \{v_1, v_2, \dots, v_n\}$ and $X = \{x_1, x_2, \dots, x_n\}$.

A graph G and its Mycielskian graph $\mu(G)$ are depicted in Figure 1 and 2.

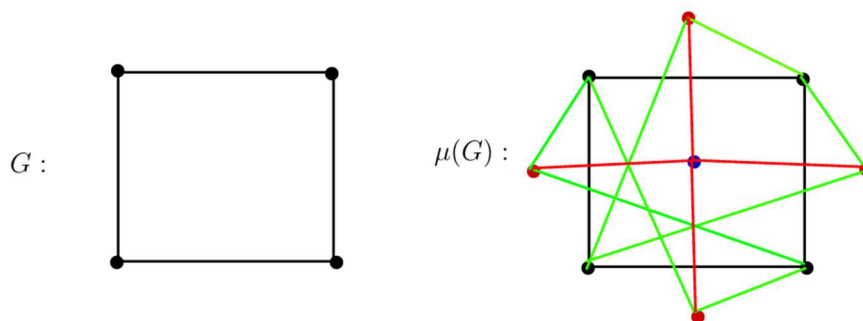
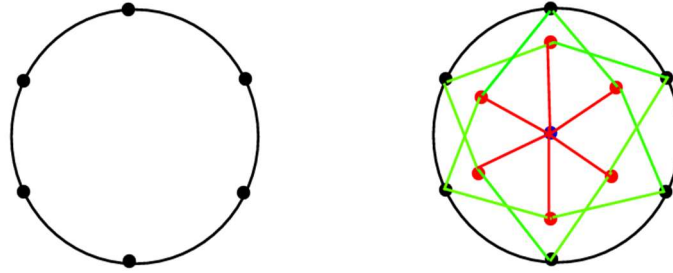


Fig. 1. A Graph G and its Mycielskian graph $\mu(G)$

Fig. 2. A graph G and its Mycielskian graph $\mu(G)$

By the construction of Mycielskian graph $\mu(G)$ we can see that the vertex set of $\mu(G)$ contains $2n + 1$ vertices. The vertices indicating black color are the original vertices of G and the vertices indicating by red color and blue color are the newly formed vertices in $\mu(G)$. Similarly, the edges indicating black color are the original edges of G and the edges indicating green and blue colors are the newly formed edges in $\mu(G)$.

3 Results

Theorem 1. Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$. If μ is mycielskian graph of G . Then the average degree-distance index of μ is given by

$$DD^a(\mu(G)) = \frac{1}{n(n-1)}[4m(n+4) - M_1(G)] + \frac{3(n+1)}{(n-1)} + 2[2(\frac{m}{n} + 1) + DD^a(G)]$$

Proof. Let J be a graph of order n and size m . The mycielskian graph $\mu(J)$ of J is of order $(2n + 1)$ and size $(2m + n)$. Now we have the following observations regarding the distance between the pair of vertices in $\mu(J)$:

- Suppose $u = x$ and $v = x_i$ then $d_\mu(u, v) = 1$
- Suppose $u = x$ and $v = v_i$ then $d_\mu(u, v) = 2$
- Suppose $u = x_i$ and $v = x_j$ then $d_\mu(u, v) = 2$
- Suppose $u = v_i, v = v_j$ and $d(v_i, v_j) \leq 3$ then $d_\mu(u, v) = d_i(v_i, v_j)$
- Suppose $u = v_i, v = v_j$ and $d(v_i, v_j) \geq 4$ then $d_\mu(u, v) = 4$

- Suppose $u = v_i$, $v = x_j$ and $i = j$ then $d_\mu(u, v) = 2$
- Suppose $u = x_i$, $v = x_j$, $i \neq j$ and $d_t(v_i, v_j) \leq 2$ then $d_\mu(u, v) = d_t(v_i, v_j)$
- Suppose $u = v_i$, $v = x_j$, $i \neq j$ and $d_t(v_i, v_j) \geq 3$ then $d_\mu(u, v) = 3$ With these information, we proceed for the following cases:

Case 1. If $u = x$ and $v \in X$ then

$$\begin{aligned} \sum_{i=1}^n d_\mu^a(x, x_i)[deg_\mu(x) + deg_\mu(x_i)] &= \frac{1}{n(n-1)}[n(n+1) + 2m] \\ &= \frac{(n+1)}{(n-1)} + \frac{2m}{n(n-1)} \end{aligned}$$

Case 2. If $u = x$ and $v \in V$ then

$$\begin{aligned} \sum_{i=1}^n d_\mu^a(x, v_i)[deg_\mu(x) + deg_\mu(v_i)] &= \frac{1}{n(n-1)}[2(n^2 + 4m)] \\ &= \frac{(2n)}{(n-1)} + \frac{8m}{n(n-1)} \end{aligned}$$

Case 3. If $\{u, v\} \subseteq X$ then

$$\begin{aligned} \sum_{\{x_i, x_j\} \subseteq X} d_\mu^a(x_i, x_j)[deg_\mu(x_i) + deg_\mu(x_j)] &= \frac{1}{n(n-1)}[2n^2 - 2n + 4(n-1)m] \\ &= 2\left(\frac{(2m)}{(n)} + 1\right) \end{aligned}$$

Case 4.

If $\{u, v\} \subseteq V$ then

$$\sum_{\{v_i, v_j\} \subseteq V} d_\mu^a(v_i, v_j)[deg_\mu(v_i) + deg_\mu(v_j)] = 2DD^a(G)$$

Case 5. If $u = v_i$ and $v = x_i$, $1 \leq i \leq n$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(v_i, x_i)[deg_{\mu}(v_i) + deg_{\mu}(x_i)] &= \frac{1}{n(n-1)}[2n + 12m] \\ &= \frac{(2)}{(n-1)} + \frac{12m}{n(n-1)} \end{aligned}$$

Case 6. If $u = v_i$ and $v = x_j$, $i \neq j$ then,

$$\begin{aligned} \sum_{\{v_i, x_j\} \subseteq V(\mu(G))} d_{\mu}^a(v_i, x_j)[deg_{\mu}(v_i) + deg_{\mu}(x_j)] &= \frac{1}{n(n-1)}[2n(n-1) + 2m(2n-3) - M_1(G)] \\ &= 2 + \frac{(2m(2n-3))}{n(n-1)} - \frac{M_1(G)}{n(n-1)} \end{aligned}$$

By combining the cases (1) to (6), we get the required result

Theorem 2. Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$. If μ is mycilskian graph of G and $\bar{\mu}$ is the compliment of the mycilskian graph. Then the average degree distance index of $\bar{\mu}(G)$ is given by

$$DD^a(\bar{\mu}(G)) = 2n + \frac{144m + 31n - 9}{(n-1)} - \frac{48n^2}{(n-1)} - \frac{2m^2}{n(n-1)} + \frac{16mn - 36m - 28M_1(G)}{n(n-1)}$$

Proof. Let G be a graph of order n and size m . The mycilskian graph $\mu(G)$ of G is of order $(2n + 1)$ and size $(2m + n)$. Now we have the following observations regarding the distance between the pair of vertices in $\mu(G)$:

- Suppose $u = x$ and $v = x_i$ then $d_{\mu}(u, v) = 1$
- Suppose $u = x$ and $v = v_i$ then $d_{\mu}(u, v) = 1$
- Suppose $u = x_i$, $v = x_j$ and $d_t(v_i, v_j) > 1$ then $d_{\mu}(u, v) = 1$
- Suppose $u = v_i$, $v = v_j$ and $d(v_i, v_j) = 1$ then $d_{\mu}(u, v) = 2$
- Suppose $u = v_i$, $v = x_j$ and $i = j$ then $d_{\mu}(u, v) = 1$
- Suppose $u = x_i$, $v = x_j$, $i \neq j$ and $d_t(v_i, v_j) \geq 1$ then $d_{\mu}(u, v) = 1$
- Suppose $u = v_i$, $v = x_j$, $i \neq j$ and $d_t(v_i, v_j) = 1$ then $d_{\mu}(u, v) = 2$

With these information, we proceed for the following cases:

Case 1. If $u = x$ and $v \in X$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(x, x_i)[deg_{\mu}(x) + deg_{\mu}(x_i)] &= \frac{1}{n(n-1)}[6n^2 - 2n - 4m] \\ &= \frac{(2(3n-1))}{(n(n-1))} - \frac{4m}{n(n-1)} \end{aligned}$$

Case 2. If $u = x$ and $v \in V$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(x, v_i)[deg_{\mu}(x) + deg_{\mu}(v_i)] &= \frac{1}{n(n-1)}[3n^2 - 4m] \\ &= \frac{(3n)}{(n-1)} - \frac{4m}{n(n-1)} \end{aligned}$$

Case 3. If $\{u, v\} \subseteq X$ then

$$\begin{aligned} \sum_{\{x_i, x_j\} \subseteq X} d_{\mu}^a(x_i, x_j)[deg_{\mu}(x_i) + deg_{\mu}(x_j)] &= \frac{1}{n(n-1)}[4n^2 - 2n - 2m(m-1)] \\ &= \frac{(2(2n-1))}{(n-1)} - \frac{2m(m-1)}{n(n-1)} \end{aligned}$$

Case 4. If $\{u, v\} \subseteq V$ then

$$\begin{aligned} \sum_{\{v_i, v_j\} \subseteq V} d_{\mu}^a(v_i, v_j)[deg_{\mu}(v_i) + deg_{\mu}(v_j)] &= \frac{1}{n(n-1)}[2n^2(n-1) + 4m - 2M_1(G)] \\ &= 2n + \frac{4m}{n(n-1)} - \frac{2M_1(G)}{n(n-1)} \end{aligned}$$

Case 5. If $u = v_i$ and $v = x_i$, $1 \leq i \leq n$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(v_i, x_i)[deg_{\mu}(v_i) + deg_{\mu}(x_i)] &= \frac{1}{n(n-1)}[4n^2 - n - 6m] \\ &= \frac{(4n)}{(n-1)} - \frac{1}{n-1} - \frac{6m}{n(n-1)} \end{aligned}$$

$$\begin{aligned}
\sum_{\{v_i, x_j\} \subseteq V(\mu(G))} d_{\mu}^a(v_i, x_j) [deg_{\mu}(v_i) + deg_{\mu}(x_j)] &= \frac{1}{n(n-1)} [4m(4n-1)] \\
&- \frac{6}{n(n-1)} [5M_1(G) + 8n^3 - 3n^2] \\
&- \frac{6}{n(n-1)} [24mn + 4m + n] \\
&= \frac{4m(4n-1)}{n(n-1)} \\
&- \frac{30M_1(G)}{n(n-1)} - \frac{48n^2}{n-1} \\
&+ \frac{18n}{n-1} + \frac{144m}{n-1} - \frac{24m}{n(n-1)} - \frac{6}{n-1}
\end{aligned}$$

By combining the cases (1) to (6), we get the required result.

Theorem 3. Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$. If μ is mycilskian graph of G . Then the average Gutman index of μ is given by

$$GI^a(\mu(G)) = \frac{(10m+n)}{(n-1)} + \frac{1}{n(n-1)} [4m^2 + 8m + 3M_2(G)] + 6GI^a + 2DD^a(G)$$

Proof. Let G be a graph of order n and size m . The mycilskian graph $\mu(G)$ of G is of order $(2n+1)$ and size $(2m+n)$. Now we have the following observations regarding the distance between the pair of vertices in $\mu(G)$:

- Suppose $u = x$ and $v = x_i$ then $d_{\mu}(u, v) = 1$
- Suppose $u = x$ and $v = v_i$ then $d_{\mu}(u, v) = 2$
- Suppose $u = x_i$ and $v = x_j$ then $d_{\mu}(u, v) = 2$
- Suppose $u = v_i, v = v_j$ and $d(v_i, v_j) \leq 3$ then $d_{\mu}(u, v) = d_t(v_i, v_j)$
- Suppose $u = v_i, v = v_j$ and $d(v_i, v_j) \geq 4$ then $d_{\mu}(u, v) = 4$
- Suppose $u = v_i, v = x_j$ and $i = j$ then $d_{\mu}(u, v) = 2$
- Suppose $u = x_i, v = x_j, i \neq j$ and $d_t(v_i, v_j) \leq 2$ then $d_{\mu}(u, v) = d_t(v_i, v_j)$
- Suppose $u = v_i, v = x_j, i \neq j$ and $d_t(v_i, v_j) \geq 3$ then $d_{\mu}(u, v) = 3$ With these information, we

proceed for the following cases:

Case 1. If $u = x$ and $v \in X$ then

$$\begin{aligned}\sum_{i=1}^n d_{\mu}^a(x, x_i)[deg_{\mu}(x) \times deg_{\mu}(x_i)] &= \frac{1}{n(n-1)}[n^2 + 2mn] \\ &= \frac{(n)}{(n-1)} + \frac{2m}{(n-1)} \\ &= \frac{(2m+n)}{(n-1)}\end{aligned}$$

Case 2. If $u = x$ and $v \in V$ then

$$\begin{aligned}\sum_{i=1}^n d_{\mu}^a(x, v_i)[deg_{\mu}(x) \times deg_{\mu}(v_i)] &= \frac{1}{n(n-1)}[8mn] \\ &= \frac{(8m)}{(n-1)}\end{aligned}$$

Case 3. If $\{u, v\} \subseteq X$ then

$$\begin{aligned}\sum_{\{x_i, x_j\} \subseteq X} d_{\mu}^a(x_i, x_j)[deg_{\mu}(x_i) \times deg_{\mu}(x_j)] &= \frac{1}{n(n-1)}[n(n-1) + 4(n-1)m + 4m^2 - M_1(G)] \\ &= 1 + \frac{4m}{n} + \frac{4m^2 - M_1(G)}{n(n-1)}\end{aligned}$$

Case 4. If $\{u, v\} \subseteq V$ then

$$\sum_{\{v_i, v_j\} \subseteq V} d_{\mu}^a(v_i, v_j)[deg_{\mu}(v_i) \times deg_{\mu}(v_j)] = 4GI^a(G)$$

Case 5. If $u = v_i$ and $v = x_i$, $1 \leq i \leq n$ then

$$\begin{aligned}\sum_{i=1}^n d_{\mu}^a(v_i, x_i)[deg_{\mu}(v_i) \times deg_{\mu}(x_i)] &= \frac{1}{n(n-1)}[4(2m + M_1(G))] \\ &= \frac{8m + 4M_1(G)}{n(n-1)}\end{aligned}$$

Case 6. If $u = v_i$ and $v = x_j$, $i \neq j$ then

$$\sum_{\{v_i, x_j\} \subseteq V(\mu(G))} d_{\mu}^a(v_i, x_j) [deg_{\mu}(v_i) \times deg_{\mu}(x_j)] = DD^a(G) + 2GI^a(G)$$

By combining the cases (1) to (6), we get the required result. Theorem 4. Let $G = (V, E)$ be a graph with $|V| = \bar{n}$ and $|E| = m$. If μ is mycilskian graph of G and (μ) is the compliment of the mycilskian graph. Then the average Gutman index of μ is given by

$$GI^a(\bar{\mu}(G)) = \frac{1}{n(n-1)} [2m^2 + 4m - 12mn + \frac{3}{2}M_1(G) + 4M_2(G)] + \frac{1}{n-1} [4n(2n-1) + 2n^2 - 8m + 4(mn - M_1(G))] + \frac{(2n-1)^2}{2(n-1)} - \frac{2m(2n-1)}{n}$$

Proof. Let G be a graph of order n and size m . The mycilskian graph $\mu(G)$ of G is of order $(2n+1)$ and size $(2m+n)$. Now we have the following observations regarding the distance between the pair of vertices in $\mu(G)$:

- Suppose $u = x$ and $v = x_i$ then $d_{\mu}(u, v) = 1$
 - Suppose $u = x$ and $v = v_i$ then $d_{\mu}(u, v) = 1$
 - Suppose $u = x_i$, $v = x_j$ and $d_t(v_i, v_j) > 1$ then $d_{\mu}(u, v) = 1$
 - Suppose $u = v_i$, $v = v_j$ and $d(v_i, v_j) = 1$ then $d_{\mu}(u, v) = 2$
 - Suppose $u = v_i$, $v = x_j$ and $i = j$ then $d_{\mu}(u, v) = 1$
 - Suppose $u = x_i$, $v = x_j$, $i \neq j$ and $d_t(v_i, v_j) \geq 1$ then $d_{\mu}(u, v) = 1$
 - Suppose $u = v_i$, $v = x_j$, $i \neq j$ and $d_t(v_i, v_j) = 1$ then $d_{\mu}(u, v) = 2$
- With these information, we proceed for the following cases:

Case 1. If $u = x$ and $v \in X$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(x, x_i) [deg_{\mu}(x) \times deg_{\mu}(x_i)] &= \frac{1}{n(n-1)} [2n(2n-1) - 2m] \\ &= \frac{2[(2n-1)n - 2m]}{(n-1)} \end{aligned}$$

Case 2. If $u = x$ and $v \in V$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(x, v_i)[deg_{\mu}(x) \times deg_{\mu}(v_i)] &= \frac{1}{n(n-1)}[2n(n^2 - 2m)] \\ &= \frac{2(n^2 - 2m)}{(n-1)} \end{aligned}$$

Case 3. If $\{u, v\} \subseteq X$ then

$$\begin{aligned} \sum_{\{x_i, x_j\} \subseteq X} d_{\mu}^a(x_i, x_j)[deg_{\mu}(x_i) \times deg_{\mu}(x_j)] &= \frac{1}{n(n-1)}[n(2n-1)^2 - (2n-1)(n-1) + 2m^2 \\ &\quad - \frac{1}{2}M_1(G)] \\ &= \frac{(2n-1)^2}{2(n-1)} - \frac{2m(2n-1)}{n} \\ &\quad + \frac{1}{n(n-1)}[2m^2 - \frac{1}{2}M_1(G)] \end{aligned}$$

Case 4. If $\{u, v\} \subseteq V$ then

$$\begin{aligned} \sum_{\{v_i, v_j\} \subseteq V} d_{\mu}^a(v_i, v_j)[deg_{\mu}(v_i) \times deg_{\mu}(v_j)] &= \frac{1}{n(n-1)}[4n^2m - 4nM_1(G) + 4M_2(G)] \\ &= \frac{4(mn - M_1(G))}{(n-1)} + \frac{4M_2(G)}{n(n-1)} \end{aligned}$$

Case 5. If $u = v_i$ and $v = x_i$, $1 \leq i \leq n$ then

$$\begin{aligned} \sum_{i=1}^n d_{\mu}^a(v_i, x_i)[deg_{\mu}(v_i) \cdot deg_{\mu}(x_i)] &= \frac{1}{n(n-1)[2n^2(2n-1) - (6n-2)2m + 2M_1(G)]} \\ &= \frac{2n(2n-1)}{(n-1)} - \frac{(3n-1)4m - 2M_1(G)}{n(n-1)} \end{aligned}$$

Case 6. If $u = v_i$ and $v = x_j$, $i \neq j$ then

$$\begin{aligned} \sum_{\{v_i, x_j\} \subseteq V(\mu(G))} d_{\mu}^a(v_i, x_j)[deg_{\mu}(v_i) \cdot deg_{\mu}(x_j)] &= \frac{1}{n(n-1)} \left[[2n(2n-1)2m - 2nM_1(G)] \right. \\ &\quad - [2(2n-1)M_1(G) + 4M_2(G)] \\ &\quad \left. + \sum_{v_i, v_j \subseteq V} 2(2n - 2deg_G(v_i))(2n - 1 - deg_G(v_i)) \right] \end{aligned}$$



By combining the cases (1) to (6), we get the required result. $\square$

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