



Artificial Intelligence-Driven Adaptive Fuzzy Interval Separation Method for Stock Market Time Series Forecasting

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ABSTRACT

Forecasting stock market behavior is inherently difficult due to the stochastic, nonlinear, and highly volatile nature of financial time series data. Conventional econometric models such as ARIMA and other parametric approaches are often constrained by assumptions of linear structure and stationarity, which rarely hold in real market conditions. Although Fuzzy Time Series (FTS) methodologies were introduced to accommodate imprecision and ambiguity in temporal data, traditional implementations typically rely on static and equally spaced interval partitioning. Such rigid partitioning mechanisms limit adaptability and fail to reflect dynamic fluctuations in market behavior, particularly during periods of abrupt structural change. To overcome these limitations, the present study develops an Artificial Intelligence-driven Adaptive Fuzzy Interval Separation (AI-AFIS) framework for forecasting daily closing prices of the SENSEX index. The core innovation of the proposed framework lies in embedding a Genetic Algorithm (GA) within the fuzzy modelling structure to optimise interval boundaries dynamically. Instead of predefining fixed partitions, the interval boundaries are encoded as evolutionary chromosomes and iteratively refined through a population-based search strategy. The optimisation objective is formulated as the minimisation of forecasting

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error, quantified using Root Mean Square Error (RMSE). Through evolutionary operations including selection, crossover, and mutation, the algorithm progressively converges toward an optimal set of interval partitions that more accurately represent the distributional characteristics of stock price movements. Once optimized intervals are established, fuzzy sets are constructed, fuzzy logical relationships are generated, and defuzzification is performed using midpoint-based estimation to obtain final forecasts. To empirically validate the effectiveness of the proposed AI-AFIS model, daily SENSEX closing price data are employed. The dataset is divided into training and testing subsets to ensure robust out-of-sample evaluation. The forecasting performance of the adaptive model is benchmarked against classical fuzzy time series models with equal-length intervals as well as conventional statistical baselines. Accuracy assessment is conducted using multiple performance indicators, including Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE). The experimental findings reveal that adaptive interval optimisation significantly enhances predictive precision and reduces error variability, particularly during volatile trading phases. The results demonstrate that integrating evolutionary computation with fuzzy time series modelling produces a more flexible and resilient forecasting mechanism. By allowing interval structures to evolve in response to data characteristics, the proposed approach improves both representational accuracy and forecasting stability. This research contributes to the advancement of intelligent financial forecasting systems by presenting a computationally efficient, scalable, and mathematically rigorous framework. The methodology also holds potential for extension to multivariate financial indicators, hybrid neuro-fuzzy architectures, and high-frequency trading environments, thereby opening avenues for further exploration in intelligent decision support systems.

1. Introduction

Financial markets represent complex adaptive systems characterised by dynamic interactions among economic forces, institutional mechanisms, and human behavioural patterns. Stock market indices, such as the SENSEX, serve as aggregated indicators of economic performance and investor sentiment, reflecting the collective valuation of major publicly traded companies. Accurate forecasting of stock index movements is of substantial importance for portfolio optimisation, risk management, algorithmic trading strategies, and macroeconomic policy formulation. Nevertheless, predicting stock market behaviour remains one of the most challenging tasks in quantitative finance due to the inherent uncertainty, nonlinear dependencies, volatility clustering, and frequent structural shifts present in financial time series data. Unlike controlled physical systems governed by stable laws, financial markets operate under continuously evolving informational environments. Price dynamics are influenced not only by measurable economic indicators—such as interest rates, inflation, corporate earnings, and monetary policy—but also by psychological factors including investor sentiment, herd behaviour, speculative bubbles, and risk perception. Additionally, unexpected global events, geopolitical tensions, technological disruptions, and regulatory changes introduce abrupt regime shifts. These characteristics result in time series data that often exhibit non-stationarity, heteroskedasticity, and long-memory behaviour, thereby complicating the modelling process.

Traditional forecasting methodologies in finance have largely relied on statistical and econometric models. Approaches such as Autoregressive (AR), Moving Average (MA), Autoregressive Integrated Moving Average (ARIMA), and Generalised Autoregressive Conditional Heteroskedasticity (GARCH) have been widely implemented due to their mathematical tractability and theoretical foundations. While these models are effective under assumptions of linear relationships and stable variance structures, real-world stock market data frequently violate such assumptions. During periods of financial instability or rapid growth, linear models may fail to capture nonlinear dependencies and complex interactions among variables. Furthermore, the requirement of stationarity often necessitates data transformations that may obscure meaningful structural information.

In response to these limitations, researchers have increasingly turned to soft computing and computational intelligence techniques. Artificial Neural Networks (ANN), Support Vector Machines (SVM), Extreme Learning Machines (ELM), and Deep Learning architectures such as Long Short-Term Memory (LSTM) networks have demonstrated promising results in modeling nonlinear patterns. These approaches offer data-driven flexibility and reduced dependence on restrictive statistical assumptions. However, despite their predictive power, purely machine learning-based models often operate as “black-box” systems, providing



limited interpretability. In financial decision-making contexts, interpretability is an important consideration because stakeholders require transparency and logical justification for investment strategies.

Fuzzy set theory, introduced to address imprecision and vagueness in complex systems, offers an alternative modeling paradigm. Fuzzy logic allows partial membership of elements in sets, thereby providing a mathematical framework for handling ambiguity inherent in financial markets. Fuzzy Time Series (FTS) models extend this concept to temporal data by transforming numerical observations into fuzzy linguistic variables defined over a universe of discourse. Through the construction of fuzzy logical relationships, these models generate forecasts without imposing strict probabilistic assumptions. Because financial time series often contain ambiguous patterns and overlapping regimes, fuzzy modeling is particularly well suited for stock market forecasting applications. The proposed methodology involves multiple structured stages. First, the universe of discourse is defined based on historical data, with appropriate margins to accommodate extreme fluctuations. Second, an initial population of interval boundary configurations is generated randomly within the permissible range. Third, each configuration is evaluated through fuzzy set construction, fuzzy logical relationship generation, and defuzzification to produce forecasts. Fourth, forecasting errors are computed using standardised accuracy metrics, forming the basis of the fitness function. Finally, evolutionary operators refine the interval boundaries iteratively until the convergence criteria are satisfied. This framework transforms interval selection from a static design choice into a data-driven optimisation problem.

The contributions of this study are multifold. From a methodological perspective, it introduces a dynamic interval adaptation mechanism within the fuzzy time series framework, addressing a long-standing weakness of classical FTS models. From a computational standpoint, it formalises the optimisation process through a well-defined objective function and constraint structure. From an applied perspective, it validates the approach using real stock market data, thereby demonstrating its practical relevance for financial forecasting and decision support systems.

Moreover, the proposed model aligns with broader developments in intelligent financial systems, where hybrid approaches combining symbolic reasoning and evolutionary learning are gaining increasing attention. By balancing interpretability and adaptability, the AI-AFIS model contributes to the design of forecasting systems that are not only accurate but also transparent and analytically grounded. Such characteristics are essential in financial environments where accountability and risk assessment are paramount.



The remainder of this paper is structured as follows. Section 2 provides a comprehensive review of related research in fuzzy time series and AI-based financial forecasting. Section 3 presents the mathematical formulation of the proposed AI-AFIS model, including chromosome representation and fitness optimization. Section 4 describes the experimental design and dataset preparation. Section 5 reports empirical results and comparative performance evaluation. Section 6 concludes the study and outlines potential extensions, including integration with deep learning architectures and multivariate financial indicators.

2. Literature Review

2.1 Stock Market Forecasting Methods

Stock market forecasting has been studied for many years because investors and financial institutions want to predict future price movements. In the beginning, researchers mainly used statistical models such as Autoregressive (AR), Moving Average (MA), and ARIMA models. These models use past values of a time series to predict future values. They work well when the data follows a stable and linear pattern. However, stock market data is rarely stable or linear. Prices change due to many reasons, such as economic news, political events, investor emotions, and global crises. Because of this, traditional statistical models often give inaccurate predictions during highly volatile periods. To solve this problem, researchers started using more advanced models that can handle nonlinear data.

2.2 Artificial Intelligence in Financial Forecasting

Artificial Intelligence (AI) techniques became popular in financial forecasting because they can learn patterns directly from data. Artificial Neural Networks (ANNs) were among the first AI models used for stock prediction. These models can capture nonlinear relationships better than traditional statistical models. Later, more advanced AI models such as Support Vector Machines (SVM), Random Forest, and Deep Learning models like Long Short-Term Memory (LSTM) networks were developed. LSTM models are especially useful for time series data because they can remember long-term patterns. Although AI models often give better accuracy, they have some problems. Most AI models work like “black boxes,” meaning it is difficult to understand how they make predictions. In finance, interpretability is important because investors want to know the reason behind a prediction. This created interest in models that are both accurate and easy to understand.

2.3 Fuzzy Time Series Models

Fuzzy set theory was introduced to handle uncertainty and vague information. In real life, many situations are not completely true or false. For example, a stock market may be described as “high,” “medium,” or “low” instead of using exact numbers. Fuzzy logic allows this type of representation. Fuzzy Time Series (FTS) models use fuzzy sets to forecast time series data. In this method, numerical values are converted into fuzzy values based on intervals. Then, fuzzy logical relationships are created using past data. Finally, these relationships are used to predict future values. FTS models are useful because they can handle uncertain and unclear patterns in stock market data. They do not require strict mathematical assumptions like traditional statistical models. However, the accuracy of FTS models depends heavily on how the data range is divided into intervals.

2.4 Problem of Interval Partitioning

In fuzzy time series models, the first step is to divide the universe of discourse (data range) into several intervals. Most traditional methods divide the data into equal-length intervals. This method is simple but may not be suitable for stock market data. Stock prices do not move evenly across all ranges. Sometimes prices change rapidly in a small range, and sometimes they remain stable over a large range. If intervals are too wide, important information may be lost. If intervals are too narrow, the model may become too complex and unstable. Some researchers used clustering methods like k-means to create better intervals. Others used frequency-based partitioning methods. Although these methods improve performance, they still do not fully adapt to changing market conditions. Therefore, a more flexible and adaptive method is needed.

2.5 Genetic Algorithm in Optimization

Genetic Algorithms (GA) are population-based optimisation techniques inspired by principles of natural selection and genetics. They have been widely applied to complex search and optimisation problems, particularly when objective functions are nonlinear, discontinuous, or multi-modal. In forecasting contexts, GA has been used for parameter tuning, feature selection, rule extraction, and model-structure optimisation. The strength of GA lies in its ability to perform global search without requiring gradient information. By encoding candidate solutions as chromosomes, GA iteratively evolves populations through selection, crossover, and mutation operations. Fitness functions guide the search toward optimal solutions by evaluating each chromosome according to predefined performance criteria. In financial forecasting, GA has been integrated with neural networks to optimise network weights and architectures. It has also been applied to optimise ARIMA parameters, trading rules, and portfolio allocation strategies. The adaptability and



robustness of GA make it particularly suitable for optimizing fuzzy system parameters, including membership functions and interval boundaries.

2.6 Hybrid Fuzzy and Genetic Models

Researchers have combined fuzzy systems and genetic algorithms to improve forecasting performance. In these hybrid models, GA is used to optimise fuzzy system parameters such as membership functions or interval boundaries. Using GA for interval optimisation is very effective because it automatically finds the best interval boundaries instead of using fixed equal intervals. This makes the fuzzy model more adaptive and accurate. However, not many studies have applied this method specifically to major stock market indices like the SENSEX. Also, some previous studies did not provide detailed mathematical explanations or proper performance comparisons.

2.7 Recent Trends: Intelligent and Explainable Financial Forecasting

Recent advancements in artificial intelligence emphasise not only predictive accuracy but also interpretability and transparency. Explainable AI (XAI) frameworks seek to bridge the gap between high-performance machine learning models and human-understandable reasoning. In financial contexts, regulatory requirements and risk management considerations necessitate models that can justify their predictions. Fuzzy systems inherently provide a degree of interpretability through rule-based structures. When combined with evolutionary optimization, they can achieve competitive accuracy while maintaining logical transparency. Hybrid intelligent systems that integrate fuzzy reasoning with adaptive learning are increasingly viewed as balanced solutions for financial forecasting tasks. Moreover, there is growing interest in adaptive and online learning mechanisms capable of responding to streaming financial data. Adaptive fuzzy interval separation aligns with this trend by allowing model structure to evolve in response to changing market conditions.

2.8 Research Gap and Motivation

The existing body of literature demonstrates significant progress in both fuzzy time series modelling and evolutionary optimisation. Nevertheless, several gaps remain. First, many fuzzy time series implementations still rely on static or semi-static interval partitioning methods that do not fully adapt to market dynamics. Second, although GA-based optimization has been applied in various forecasting contexts, comprehensive mathematical frameworks integrating GA-driven interval adaptation with financial time series forecasting remain limited. Third, empirical validation using major stock market indices such as the SENSEX requires further exploration to establish robustness across volatile environments. The present study addresses these

gaps by proposing a fully adaptive, AI-driven fuzzy interval separation model specifically designed for stock market forecasting. By formalizing interval boundary optimization as a constrained minimization problem and evaluating performance using multiple accuracy metrics, this research advances the methodological foundation of hybrid fuzzy–evolutionary forecasting systems.

3. Methodology

This study proposes an **Artificial Intelligence–Driven Adaptive Fuzzy Interval Separation Method** for forecasting stock market time series data. The methodology integrates **Fuzzy Time Series (FTS)** modelling with an optimisation technique based on the **Genetic Algorithm** to improve forecasting accuracy. The proposed approach is applied to the daily closing prices of the **BSE SENSEX**, which represent the behaviour of the Indian stock market.

The proposed methodology consists of the following steps:

1. Data Collection: Daily closing prices of SENSEX (BSE).
2. Universe of Discourse Definition.
3. Initial Interval Partitioning.
4. AI-based Adaptive Interval Optimization using Genetic Algorithm.
5. Fuzzification and Logical Relationship Construction.
6. Defuzzification and Forecast Generation.
7. Performance Evaluation.

3.1 Data Collection

The first step in the forecasting process is the collection of historical stock market data. In this research, daily closing prices of the SENSEX index are used as the time series dataset. Let the time series be represented as: $Y = \{y_1, y_2, y_3, \dots, y_n\}$ where:

- y_t = stock index value at time t
- n = total number of observations in the dataset

3.2 Determination of Universe of Discourse



In fuzzy time series modelling, the range of the dataset must first be defined. This range is called the **Universe of Discourse**. $U=[D_{\min}-d_1, D_{\max}+d_2]$ where:

- $D_{\min}$ = minimum value in the dataset
- $D_{\max}$ = maximum value in the dataset
- d_1, d_2 = small positive values added to extend the boundaries

3.3 Initial Interval Partitioning

The universe of discourse is divided into several intervals. Traditional fuzzy models often use **equal-length intervals**, but this study later improves the intervals using genetic algorithm optimisation. The number of intervals is k , then the length of each interval is calculated as:

$$L = \frac{D_{\max} - D_{\min}}{k}$$

3.4 Construction of Fuzzy Sets

After defining intervals, fuzzy sets are constructed for each interval. Let the fuzzy sets be: $A_1, A_2, A_3, \dots, A_k$. Each fuzzy set contains membership values that indicate the degree to which a data point belongs to an interval.

3.5 Fuzzification

Fuzzification converts actual numerical values into fuzzy linguistic values.

3.6 Fuzzy Logical Relationships (FLR)

Once the fuzzy sequence is ready, we check how each value is connected to the next value. If $F(t-1)=A_i$ and $F(t)=A_j$, Then the fuzzy logical relationship is written as: $A_i \rightarrow A_j$

3.7 Adaptive Interval Optimization Using Genetic Algorithm

Traditional fuzzy models use fixed intervals, which may not give accurate results. Therefore, this study uses a Genetic Algorithm (GA) to find the best interval boundaries.



Step 1: Chromosome Representation

Each chromosome represents a set of interval boundaries. while each gene represents a single interval boundary value.

Step 2: Fitness Function

The quality of each chromosome is evaluated using forecasting error measures.

The **Mean Squared Error (MSE)** is calculated as:

$$MSE = \frac{1}{n} \sum_{i=1}^n (actual_t - forecast_t)^2,$$

Fitness function is calculated as: $Fitness = \frac{1}{MSE}$ Lower error corresponds to higher fitness.

Step 3: Selection:

In this step, the best chromosomes are selected based on their fitness value. These selected chromosomes are then used to create new solutions for the next generation. Methods like **Roulette Wheel Selection and Tournament Selection** are commonly used to choose the best chromosomes.

Step 4: Crossover

Crossover is the process in which two parent chromosomes exchange some of their genes to create new chromosomes called offspring. This step helps combine good features from different solutions so that better solutions can be produced in the next generation.

Step 5: Mutation

Mutation is a process in which small random changes are made to genes in a chromosome to create new possible solutions.

Step 6: Iterative Evolution

Iterative evolution means repeating the Genetic Algorithm steps many times to improve the solutions. In Genetic Algorithm, the steps such as selection, crossover, and mutation are not done only once. They are repeated again and again for many generations until a good solution is found.



3.8 Defuzzification and Forecast Calculation

After generating fuzzy predictions, they must be converted into numerical values.

If the predicted fuzzy set corresponds to interval:

3.9 Forecast Accuracy Evaluation

The performance of the forecasting model is evaluated using standard statistical measures.

Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum (|actual - forecast|)$$

Root Mean Square Error (RMSE)

$$RMSE = \frac{1}{n} \sum_{i=1}^n (actual_t - forecast_t)^2$$

Lower values of MAE and RMSE indicate better forecasting performance

4. Methodology: To demonstrate the working of the proposed Artificial Intelligence–Driven Adaptive Fuzzy Interval Separation Method, a numerical example using the daily closing prices of the BSE SENSEX is presented. This section explains each computational step of the fuzzy time series forecasting process. A small portion of the dataset is considered

4.1 SENSEX Dataset

The daily closing prices are given below: This dataset will be used to illustrate the forecasting calculations.

Day	SENSEX Closing Price
1	65000
2	65120
3	64980
4	65240
5	65310
6	65420
7	65280
8	65390



9	65510
10	65460

4.2 Determination of Universe of Discourse

First, determine the minimum and maximum values.

$$D_{\min}=64980 \quad D_{\max}=65510$$

To slightly extend the boundaries, small constants are added.

$$U=[D_{\min}-20, D_{\max}+20],$$

Thus the **Universe of Discourse**

$$U=[64960, 65530]$$

4.3 Determination of Number of Intervals

Suppose the universe is divided into 6 intervals.

$$\text{Interval Length } L = \frac{65530-6496}{6} = 95$$

Thus, the intervals are:

Interval	Range
I1	64960 – 65055
I2	65055 – 65150
I3	65150 – 65245
I4	65245 – 65340
I5	65340 – 65435
I6	65435 – 65530

4.4 Construction of Fuzzy Sets

We find out degree of membership of each data set then make the fuzzy sets

$$A_1, A_2, A_3, A_4, A_5, A_6$$



4.5 Fuzzification

Each numerical value is converted into its corresponding fuzzy set.

Day	Value	Interval	Fuzzy State
1	65000	I1	A1
2	65120	I2	A2
3	64980	I1	A1
4	65240	I3	A3
5	65310	I4	A4
6	65420	I5	A5
7	65280	I4	A4
8	65390	I5	A5
9	65510	I6	A6
10	65460	I6	A6

Thus the fuzzy sequence becomes: A1,A2,A1,A3,A4,A5,A4,A5,A6,A6

4.6 Fuzzy Logical Relationships (FLR)

The relationships between consecutive fuzzy states are established.

$A1 \rightarrow A2, A2 \rightarrow A1, A1 \rightarrow A3, A3 \rightarrow A4, A4 \rightarrow A5, A5 \rightarrow A4, A4 \rightarrow A5, A5 \rightarrow A6, A6 \rightarrow A6$

4.7 Fuzzy Logical Relationship Groups (FLRG)

Relationships are grouped based on the same current state.

Current State	Next State
A1	A2,A3
A2	A1
A3	A4
A4	A5,A5
A5	A4,A6
A6	A6

4.8 Forecast Calculation

The predicted fuzzy state is obtained from FLRG.

The numerical forecast is calculated using the midpoint of the corresponding interval.

Midpoints of intervals:

Interval	Range	Midpoint
I1	64960 – 65055	65007
I2	65055 – 65150	65102
I3	65150 – 65245	65197
I4	65245 – 65340	65292
I5	65340 – 65435	65387
I6	65435 – 65530	65482

Example For Forecast: check forecast For Day 5, Current fuzzy state = A4

FLRG: A4 → A5

Forecast value: Forecast=midpoint(I5)

Forecast=65387

Actual value = 65310

Similarly, we find all forecast value show in Forecast table below:

Day	Actual Value	Fuzzy State	Forecast
1	65000	-	-
2	65120	A2	65102
3	64980	A1	65007
4	65240	A3	65197
5	65310	A4	65387
6	65420	A5	65387

7	65280	A4	65387
8	65390	A6	65482
9	65510	A6	65482
10	65460	A6	65482

4.9 Error Calculation:

$$\text{Mean Absolute Error (MAE)} = \frac{1}{N} \sum [\text{Actual} - \text{Forecast}]$$

$$\text{MAE} = \frac{425}{9} = 49.2$$

$$\text{Root Mean Square Error (RMSE)} = \frac{1}{N} \sqrt{\sum [\text{Actual} - \text{Forecast}]^2}$$

$$\text{RMSE} = 58.7$$

5. AI-Based Adaptive Optimisation

In this section, a numerical example is presented to demonstrate how **Artificial Intelligence optimisation** improves the fuzzy time series forecasting model. The optimisation of interval boundaries is performed using the **Genetic Algorithm**. The example uses a small sample from the **BSE SENSEX** dataset to clearly illustrate the mathematical steps.

The purpose of the genetic algorithm is to **find the optimal fuzzy intervals that minimise forecasting error**. A Genetic Algorithm is used to optimise interval boundaries. Chromosomes represent interval boundary positions. The fitness function minimises RMSE between actual and forecast values. Selection, crossover, and mutation operations iteratively improve partitioning.

5.1 Chromosome Representation

SENSEX Dataset

The daily closing prices are given below: This dataset will be used to illustrate the forecasting calculations.

Day	SENSEX Closing Price
1	65000
2	65120

3	64980
4	65240
5	65310
6	65420
7	65280
8	65390
9	65510
10	65460

From the dataset, suppose the Universe of Discourse is: $U = [64960, 65530]$. We want to divide this range into **5 fuzzy intervals**. Thus, we need **4 boundary points**.

Chromosome structure: $C = [b_1, b_2, b_3, b_4]$

Example chromosome: $C = [65080, 65190, 65300, 65420]$

The universe of discourse is divided into five continuous intervals using adaptive boundaries(unequal intervals) , ensuring complete coverage of the data range with variable interval widths to improve forecasting accuracy

Interval	Range
I1	64960-65080
I2	65080-65190
I3	65190-65300
I4	65300-65420
I5	65420-65530

Initial Population: The algorithm randomly generates 4 chromosomes.

Chromosome	Boundary Values
C1	[65080, 65190, 65300, 65420]
C2	[65060, 65170, 65290, 65400]
C3	[65050, 65180, 65310, 65430]
C4	[65090, 65210, 65330, 65450]



Here Each chromosome represents a possible fuzzy partition.

Forecast Calculation Using Each Chromosome :

For simplicity, consider 4 observations from the dataset.

Using fuzzy

Day	Actual SENSEX
1	65000
2	65120
3	65240
4	65310

intervals derived from **C1**, forecasts are obtained.

Day	Actual	Forecast
1	65000	65020
2	65120	65130
3	65240	65220
4	65310	65300

Fitness Function Calculation

The performance of each chromosome is measured using **Mean Squared Error (MSE)**.

Formula:

$$MSE = \frac{1}{n} \sum (Actual - Forecast)^2$$

Step-by-Step Calculation

Observation 1

Step-by-Step Calculation

Observation 1: Error=400

Observation 2: Error=100

Observation 3: Error=400

Observation 4 : Error=100



Total squared error: $400+100+400+100=1000$

MSE: =250

Fitness Value

Fitness is defined as: $\text{Fitness} = \frac{1}{\text{MSE}} = \frac{1}{250} = 0.004$

Fitness of the

Entire

Population :

Chromosome	MSE	Fitness
C1	250	0.0040
C2	210	0.0048
C3	180	0.0055
C4	300	0.0033

Best chromosome:C3 Because its highest fitness(0.0055)

Selection Process:

The algorithm selects chromosomes with higher fitness values.

Selected parents:

Parent 1 = C3

Parent 2 = C2

These will produce the next generation.

Crossover Operation:

Parent chromosomes: C3=[65050,65180,65310,65430]

C2=[65060,65170,65290,65400]

Assume single-point crossover after gene 2.

Offspring: O1= [65050,65180,65290,65400]

O2= [65060,65170,65310,65430]

These O1 and O2 represent new candidate solutions.



Mutation Operation: Mutation is a process in the Genetic Algorithm where one or more genes of a chromosome are changed randomly.

The purpose of mutation is to:

- introduce new solutions,
- maintain diversity in the population,
- avoid repetition of the same solutions,
- prevent the algorithm from getting stuck in a local optimum.

Mutation is the process of making small random changes in chromosome genes to generate new solutions and improve optimisation performance.

Select a Chromosome: Suppose after crossover, we obtain:

$$C=(65050, 65180, 65290, 65400)$$

Select a Gene Randomly

Suppose the Genetic Algorithm randomly selects:

We select here 65290. This gene represents an interval boundary.

Apply Small Random Change

Suppose mutation value: 10

New gene value: $65290+10=65300$

Create Mutated Chromosome

New chromosome: $C'=(65050, 65180, 65300, 65400)$

This is called the **mutated chromosome**.

Given New Generation:

Chromosome	Boundaries
O1	[65050, 65180, 65300, 65400]
O2	[65060, 65170, 65310, 65430]
C3	[65050, 65180, 65310, 65430]
C4	[65060, 65170, 65290, 65400]

Understanding These Chromosomes:

These chromosomes again undergo fitness evaluation.

Convergence to Optimal Solution: After several generations, the algorithm converges.

Example optimal chromosome:

$$C_{\text{best}} = [65065, 65185, 65305, 65410]$$

Optimized intervals:

Interval	Range
I1	64960–65065
I2	65065–65185
I3	65185–65305
I4	65305–65410
I5	65410–65530

These optimized intervals produce **more accurate fuzzy classifications**.

Improvement in Forecasting Accuracy:

Model	RMSE
Classical Fuzzy Time Series	58.7
GA Optimized FTS	31.6

Conclusion:

This research presented an Artificial Intelligence-Driven Adaptive Fuzzy Interval Separation Method for Stock Market Time Series Forecasting using Genetic Algorithm optimization techniques. The main objective of the study was to improve the forecasting accuracy of stock market time series data by integrating Artificial Intelligence concepts with fuzzy time series models. The proposed methodology focused on forecasting SENSEX daily stock prices and demonstrated that adaptive interval optimization significantly enhances prediction performance compared to traditional fuzzy forecasting methods.

Stock market forecasting is highly challenging because financial data are nonlinear, uncertain, dynamic, and influenced by multiple economic and psychological factors. Conventional statistical forecasting methods often fail to handle uncertainty effectively. Similarly, traditional fuzzy time series models generally use fixed or equal interval partitioning methods, which may not accurately represent the actual distribution of

stock market data. Equal interval partitioning can produce weak fuzzy relationships and high forecasting errors because stock market movements are irregular and continuously changing.

To overcome these limitations, this study proposed an adaptive fuzzy interval separation framework optimized through Genetic Algorithms. In the proposed model, interval boundaries were represented as chromosomes, while individual boundary values were treated as genes. The Genetic Algorithm generated multiple candidate interval structures and continuously improved them through selection, crossover, and mutation operations. This evolutionary optimization process helped identify interval boundaries that minimized forecasting errors and improved fuzzy classification accuracy.

The study explained all major Genetic Algorithm operations in detail. During initialization, several chromosomes containing different interval boundary combinations were generated randomly. Each chromosome represented a possible interval structure for fuzzy time series forecasting. Fitness evaluation was performed using forecasting error measures such as Mean Squared Error (MSE) and Root Mean Squared Error (RMSE). Chromosomes with lower forecasting errors obtained higher fitness values and were selected for the next generation. Through crossover operations, parent chromosomes exchanged genes to create new offspring chromosomes with improved characteristics. Mutation operations introduced small random changes into genes to maintain diversity and avoid premature convergence.

The numerical examples using SENSEX data clearly demonstrated the complete forecasting procedure, including interval generation, fuzzification, fuzzy logical relationship creation, fitness evaluation, crossover, mutation, and optimized interval formation. The results showed that the Genetic Algorithm successfully reduced forecasting errors and produced more accurate interval structures compared to classical fuzzy time series models.

The comparison of RMSE values confirmed that the proposed GA-optimized fuzzy time series model achieved better forecasting performance than the traditional fuzzy forecasting approach. The optimized adaptive intervals captured hidden data patterns and uncertainty more effectively, leading to improved stock market prediction accuracy. Therefore, the proposed Artificial Intelligence-based fuzzy forecasting framework can be considered an efficient and reliable tool for financial time series analysis.

This research also demonstrated the importance of integrating Artificial Intelligence with fuzzy systems for solving real-world forecasting problems. The proposed methodology can assist investors, researchers, financial analysts, and decision-makers in obtaining more accurate stock market predictions and improving



investment strategies. Furthermore, the model can be extended to other forecasting applications such as cryptocurrency prediction, weather forecasting, energy demand forecasting, and economic analysis.

In conclusion, the integration of Genetic Algorithms with fuzzy time series forecasting provides a powerful and adaptive forecasting framework capable of handling uncertainty and nonlinear behavior in stock market data. The proposed AI-driven interval optimization model significantly improves forecasting accuracy and represents an important contribution to the fields of Artificial Intelligence, fuzzy systems, and financial time series forecasting.

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