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A Comparative Study of Stability Mechanisms in Rotating and Stratified Incompressible Fluid Models

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ABSTRACT

Rotating and stratified incompressible fluid models describe nonlinear fluid motion influenced by Coriolis forces, density gradients, buoyancy, and viscous effects. Their stability mechanisms differ because rotation generates inertial-wave dispersion, whereas stable stratification produces restoring buoyancy forces and internal gravity waves. This study aims to compare these mechanisms and evaluate their combined influence on mathematical energy relationships, solution boundedness, regularity, and long-term behaviour. It adopts a descriptive, analytical, and comparative approach using exclusively secondary evidence from freely accessible mathematical research articles, academic publications, and recognised repositories. Purposively selected studies provide governing formulations, energy estimates, conditional stability criteria, and asymptotic findings, which are compared with attention to functional spaces, boundary conditions, and initial-data restrictions. The analysis indicates that Coriolis acceleration performs no direct work in the standard kinetic-energy balance, while buoyancy transfers energy between kinetic and potential components. Viscosity and scalar diffusivity dissipate energy, and suitable combined energy inequalities

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control solution norms under specified assumptions. Rotational dispersion and stratification-related wave dynamics influence perturbation evolution differently, while their coupling can support conditional stability or introduce additional instability mechanisms. Finite-energy weak solutions satisfy energy inequalities, whereas uniqueness and stronger regularity require further analytical conditions. Reported temporal decay estimates and perturbation thresholds depend on parameter regimes, model geometry, and admissible initial data. The study concludes that boundedness, regularity, and asymptotic convergence are distinct properties. This distinction remains particularly important for nonlinear three-dimensional coupled systems, where reliable stability comparisons require explicit mathematical assumptions rather than universal claims about the stabilising effects of rotation and stratification.

1. INTRODUCTION

Incompressible fluid models constitute an essential mathematical framework for examining nonlinear fluid motion through conservation laws and partial differential equations. Rotating systems incorporate Coriolis forces that influence fluid trajectories and inertial-wave propagation, while stratified systems involve density gradients and buoyancy forces represented through the Boussinesq approximation. Stability describes the control of disturbances, whereas instability involves their growth under specified conditions. Boundedness concerns finite solution norms, while regularity describes mathematical smoothness. Comparing rotating, stratified, and coupled models is therefore important for understanding their distinct stability mechanisms.

Fluid behaviour depends on angular velocity, buoyancy frequency, viscosity, thermal diffusivity, and initial conditions. Reynolds, Rossby, Froude, and Richardson numbers characterise competing inertial, rotational, buoyancy, and dissipative effects. Nonlinear convection, kinetic–potential energy exchange, inertial waves, and internal gravity waves further influence stability. Historically, mathematical fluid dynamics evolved from Euler's inviscid formulation to the viscous Navier–Stokes equations and subsequently incorporated rotational dynamics and Boussinesq modelling. Developments in energy methods, Sobolev spaces, perturbation theory, and nonlinear functional analysis have strengthened mathematical investigations of atmospheric and oceanic flows.

Recent research increasingly employs anisotropic Littlewood–Paley theory, dispersive estimates, and refined stability thresholds. These developments emphasise that rotation and stratification do not necessarily produce unconditional stability. Contemporary mathematical evidence includes Reynolds-number-dependent perturbation thresholds, decay estimates, and functional-space restrictions. For example, Huang et al. (2026) establish conditional nonlinear stability for perturbations bounded by $\delta Re^{(-14/15)}$, requiring a sufficiently small positive constant δ and regularity index $N \geq 11$ under their specified model assumptions.

Oxley and Kerswell (2024), in *Linear Stability of Stratified, Rotating, Viscous Plane Couette–Poiseuille Flow*, investigated the combined influence of rotation, stratification, and viscosity on linear stability. Employing linearised governing equations and numerical eigenvalue analysis, they identified shear-related, centrifugal, and stratification-modified instabilities. Their findings demonstrate that combined physical mechanisms can introduce additional unstable modes rather than universally improving stability.

Fujii and Li (2024), in *Linear and Nonlinear Stability for the 3D Stratified Boussinesq Equations with the Horizontal Viscosity and Diffusivity*, investigated long-term solution behaviour under partial dissipation. Using anisotropic Littlewood–Paley theory and mathematical decay estimates, they established precise L^1–Lp decay relationships for $2 \leq p \leq \infty$. Their results identify enhanced decay in selected solution components and demonstrate the significance of stratification-induced dispersion and anisotropic dissipation.

Huang et al. (2026), in *Stability Threshold for 3D Boussinesq Equations with Rotation near the Couette Flow and Stratified Temperature*, examined nonlinear stability near stratified Couette flow. Applying dispersive estimates, mathematical multipliers, and quasi-linearisation, they established conditional perturbation thresholds and identified enhanced dissipation and inviscid damping. Their findings demonstrate how rotation modifies dispersion and supports stability under restrictive initial-data conditions.

Nonlinear coupling among Coriolis forces, buoyancy, and viscosity complicates comparisons of fluid stability. Classical energy bounds do not independently establish unconditional regularity, while published stability criteria frequently involve different domains, initial conditions, and functional spaces. A coherent comparative interpretation is therefore required to distinguish individual stability mechanisms from their combined effects.

The study aims to comparatively examine rotational and stratification-related stability mechanisms through their mathematical formulations, energy relationships, regularity conditions, and asymptotic behaviour, with particular emphasis on distinguishing established general properties from conditional stability results.

A descriptive, analytical, and comparative research design was adopted in this study using exclusively secondary mathematical evidence without original simulations or theorem development. Relevant mathematical formulations, stability estimates, and published findings were collected from freely accessible research articles, academic publications, and recognised repositories. Purposive sampling was employed in this study. Accessible studies addressing rotating, stratified, and coupled incompressible models with clearly stated mathematical assumptions were included. Duplicate, inaccessible, unrelated, and unsupported materials were excluded. Evidence was classified according to governing equations, characteristic parameters, energy estimates, dispersive mechanisms, regularity conditions, and asymptotic behaviour. Findings were compared with consideration of boundary conditions, initial-data restrictions, and mathematical limitations.

2. MATHEMATICAL FORMULATION AND GOVERNING PROPERTIES OF ROTATING AND STRATIFIED INCOMPRESSIBLE FLUID MODELS

Mathematical Formulation of Rotating Incompressible Fluid Models

Rotating incompressible fluid models extend the classical Navier–Stokes equations by incorporating the Coriolis acceleration generated within a rotating reference frame. These equations describe momentum conservation through the combined effects of nonlinear convection, pressure gradients, viscous diffusion, and rotational forces. The incompressibility constraint requires the velocity field to remain divergence-free, ensuring mass conservation under constant reference-density assumptions.

The rotational contribution depends on angular velocity and modifies the direction of fluid motion without directly changing its kinetic energy. Unlike classical incompressible models, rotating systems exhibit inertial-wave behaviour and rotationally influenced nonlinear interactions. For example, ocean currents experience Coriolis deflection because of Earth's rotation, producing flow structures that differ from equivalent nonrotating conditions. Cheng and Sakellaris (2023) demonstrated the mathematical importance of rotational interactions through a near-resonance approximation that retains interactions between oscillatory modes in rotating–stratified systems.

Mathematical Formulation of Stratified Boussinesq Fluid Models

Stratified Boussinesq models describe fluid motion influenced by density differences resulting from temperature or salinity variations. The Boussinesq approximation treats density as approximately constant in inertial terms while retaining density variations responsible for buoyancy. Consequently, momentum conservation becomes coupled with a scalar transport equation governing buoyancy perturbations.

Stable stratification occurs when denser fluid lies beneath lighter fluid, producing restoring forces against vertical displacement. Its mathematical strength is characterised by the squared buoyancy frequency, N^2, which remains positive under stable conditions. Vertical velocity couples with buoyancy perturbations, while viscosity and thermal or scalar diffusivity influence the dissipation of disturbances. These interactions support internal gravity waves that differ fundamentally from rotation-generated inertial waves.

For example, a vertically displaced water parcel in a stably stratified lake experiences a restoring buoyancy force and may oscillate around its equilibrium position. Early et al. (2021) demonstrated the importance of separating wave and vortical motions in rotating Boussinesq systems, showing how variable stratification influences modal behaviour and nonlinear energy transfers.

Governing Characteristics of Combined Rotating–Stratified Fluid Systems

Combined rotating–stratified models incorporate both Coriolis acceleration and buoyancy forces within the incompressible momentum equation. Their mathematical formulation provides a common framework for comparing individual rotational and stratification-related mechanisms.

Equation 1: Rotating–Stratified Incompressible Momentum Equation

$$\partial u/\partial t + (u \cdot \nabla)u + f(e_3 \times u) + \nabla p = \nu \Delta u + b\, e_3$$

$$\nabla \cdot u = 0$$

Here, u represents velocity, f denotes the signed Coriolis parameter, e_3 represents the vertical unit vector, p denotes modified pressure divided by reference density, ν represents kinematic viscosity, and b denotes buoyancy perturbation. The complete stratified model additionally requires an evolution equation for buoyancy.

Rotational and buoyancy terms operate through different mathematical mechanisms. Rotation modifies horizontal momentum and supports inertial oscillations, whereas buoyancy influences vertical momentum and generates internal gravity waves. Their coupling produces inertia–gravity waves whose characteristics depend on rotational intensity and stratification strength. For example, large-scale ocean circulation involves both Earth's rotation and vertical density gradients, creating coupled motions that cannot be adequately represented through either mechanism independently.

The mathematical behaviour also depends on domain geometry, initial velocity and buoyancy distributions, and boundary conditions. Periodic domains, bounded containers, and unbounded fluid regions impose different analytical requirements. Thus, the combined formulation provides a consistent basis for comparing energy relationships, solution regularity, and conditional stability while recognising that different mathematical assumptions may produce different long-term outcomes.

3. COMPARATIVE ANALYSIS OF ROTATIONAL AND STRATIFICATION-RELATED STABILITY MECHANISMS

Rotational Stability Mechanisms and Coriolis-Induced Dispersion

Rotational stability in incompressible fluid systems depends on the interaction between Coriolis forces, nonlinear convection, and viscous dissipation. The Coriolis force modifies fluid trajectories and generates inertial waves, introducing dispersive mechanisms that can restrict nonlinear interactions under suitable mathematical conditions. Rotational intensity is characterised through angular velocity and the Rossby number, which expresses the relative importance of inertial and rotational effects. Lower Rossby numbers indicate stronger rotational influence, although they do not independently guarantee stability.

Liu and Niu (2024) established global well-posedness for three-dimensional anisotropic rotating Navier–Stokes equations under small initial-data assumptions in a critical function space. Their analysis also examined asymptotic behaviour as the Rossby number approached zero. These findings demonstrate that rotational effects can support global solution control within specifically defined mathematical frameworks.

For example, increasing angular velocity in an idealised rotating fluid system decreases its Rossby number when other characteristic quantities remain constant. This may strengthen dispersive control over certain disturbances, but nonlinear instability remains possible under different initial conditions or domain restrictions.

Buoyancy-Driven Stability and Stratification-Related Dispersion

Stratification-related stability arises from density variations and associated buoyancy forces. Stable stratification occurs when denser fluid lies below lighter fluid, generating restoring forces against vertical displacement. Conversely, gravitationally unstable density arrangements can promote overturning and convective motion. Stable stratification supports internal gravity waves, whose mathematical properties depend on buoyancy frequency and fluid geometry.

Equation 2: Buoyancy Perturbation Equation

$$\partial b/\partial t + (u \cdot \nabla)b + N^2 w = \kappa \Delta b$$

Here, b represents buoyancy perturbation, u denotes velocity, N^2 is the positive squared buoyancy frequency of a uniformly stable reference state, w represents vertical velocity, and κ denotes buoyancy diffusivity. The equation describes buoyancy transport, vertical velocity coupling, and scalar diffusion under the Boussinesq approximation.

For example, a water parcel displaced vertically in a stably stratified lake experiences restoring buoyancy, producing oscillatory motion. However, sufficiently strong velocity shear can overcome stabilising influences. The gradient Richardson number compares stratification with vertical shear. For steady, inviscid, parallel shear flows, values exceeding 1/4 everywhere provide a sufficient condition for linear stability, but this criterion does not apply universally to viscous or nonlinear systems (Parker, 2020).

Qiu and Wang (2025) established stability near hydrostatic equilibrium for a three-dimensional fractional anisotropic Boussinesq model, using appropriate energy functionals and Sobolev regularity assumptions. Their findings demonstrate that buoyancy coupling and horizontal dissipation can support controlled long-term behaviour despite restricted vertical dissipation.

Comparative Interpretation of Rotational and Stratified Stability Conditions

Rotational and stratification-related mechanisms differ in their physical origins and mathematical consequences. Rotation generates inertial waves through Coriolis effects, whereas stable stratification generates internal gravity waves through buoyancy restoration. In combined systems, these mechanisms interact to produce inertia–gravity waves. Vidal and Colin de Verdière (2024) demonstrated the significance of coupled rotational and stratification effects through mathematical analysis of inertia–gravity oscillations in geophysical vortices.

Comparative stability assessment requires consideration of Reynolds number, Rossby number, buoyancy frequency, Richardson number, and admissible initial conditions. Rotational dispersion may improve conditional solution estimates, while stable stratification can restrict vertical disturbances. Nevertheless, viscosity, shear instability, anisotropic dissipation, and nonlinear coupling can modify these effects. For example, a strongly stratified rotating ocean current may exhibit vertically constrained motion while remaining vulnerable to shear-induced instability.

Overall, the comparative evidence indicates that rotation and stratification provide distinct but interacting stability mechanisms. Their effectiveness depends on mathematical assumptions, functional-space restrictions, and flow characteristics rather than the unconditional suppression of nonlinear disturbances.

4. ENERGY ESTIMATES, SOLUTION BOUNDEDNESS AND LONG-TERM BEHAVIOUR IN COUPLED FLUID MODELS

Comparative Analysis of Energy Transfer and Dissipation

Energy analysis provides a mathematical framework for understanding boundedness and dissipation in rotating and stratified incompressible fluid systems. Kinetic energy represents the integrated squared velocity magnitude, whereas buoyancy-related potential energy describes the contribution of density perturbations under stable stratification. In rotating systems, Coriolis acceleration remains perpendicular to velocity and therefore contributes no direct work to the kinetic-energy balance. Conversely, stratified systems permit energy exchange between kinetic and buoyancy components through vertical motion.

For a uniformly stratified, unforced Boussinesq system with constant positive viscosity, diffusivity, and buoyancy frequency, the combined energy balance is expressed as follows:

Equation 3: Combined Kinetic and Buoyancy Energy Balance

$$E(t) = (1/2) \int_D [|u|^2 + b^2/N^2] \, dx$$

$$dE/dt + \nu \int_D |\nabla u|^2 \, dx$$

$$+ (\kappa/N^2) \int_D |\nabla b|^2 \, dx = 0$$

Here, $E(t)$ denotes total perturbation energy, ν represents viscosity, κ denotes buoyancy diffusivity, and N^2 represents the positive squared buoyancy frequency. The identity applies to sufficiently regular solutions under appropriate periodic or energy-compatible boundary conditions. Suitable weak solutions satisfy the corresponding energy inequality.

For example, vertical displacement of a water parcel in a stably stratified ocean transfers energy between kinetic and buoyancy components, while viscosity and diffusion reduce their combined energy. Selmi and Alanazi (2025) demonstrated the importance of coupled energy estimates in establishing global solutions and long-term boundedness for a regularised three-dimensional Boussinesq system.

Weak and Strong Solutions, Regularity and Boundedness Conditions

Weak and strong solutions differ primarily in mathematical regularity and admissible functional-space conditions. Weak solutions satisfy governing equations in a distributional sense and generally possess finite-energy bounds. Strong solutions exhibit additional smoothness, permitting more restrictive uniqueness and continuous-dependence estimates.

Sobolev spaces provide essential tools for evaluating velocity gradients, buoyancy perturbations, and nonlinear coupling. Initial-data regularity and boundary conditions significantly influence the applicability of existence and stability results. Chamorro and Mîndrilă (2024) established local regularity improvements for weak solutions of the three-dimensional Boussinesq system through partial suitability conditions.

For example, square-integrable initial velocity supports finite-energy estimates, whereas stronger Sobolev regularity may permit additional analytical control. Nevertheless, existence and uniqueness results established for regularised models cannot automatically be extended to unrestricted three-dimensional systems.

Asymptotic Behaviour and Stability of Coupled Rotational–Stratified Flows

Asymptotic stability concerns whether solutions approach an equilibrium as time increases, while boundedness requires their relevant norms to remain controlled. Viscosity and scalar diffusion contribute to dissipative behaviour, whereas rotation and stratification modify wave dispersion and nonlinear interactions.

Dong (2026) established conditional asymptotic stability for three-dimensional stratified Boussinesq equations with velocity damping, demonstrating algebraic convergence towards a stationary solution under sufficiently small perturbations in higher-order Sobolev spaces. These results illustrate the dependence of long-term behaviour on specific dissipative mechanisms and regularity assumptions.

For example, a rotating, stably stratified fluid may display decreasing disturbances through combined dissipative and dispersive effects. However, strong shear, large initial perturbations, or insufficient dissipation may prevent convergence. Therefore, stability thresholds and decay estimates must be interpreted within their prescribed mathematical settings. The comparative analysis indicates that rotational–stratification coupling can influence solution behaviour substantially, but unconditional global regularity remains unresolved for general three-dimensional nonlinear systems.

5. COMPARATIVE ANALYSIS, FINDINGS AND DISCUSSION

The comparative analysis of rotating, stratified, and combined rotating–stratified incompressible fluid models has identified significant differences in their governing equations, energy relationships, stability

mechanisms, and long-term behaviour. The findings indicate that rotational forces primarily influence fluid trajectories and wave dispersion, whereas density stratification introduces buoyancy-driven motion and additional energy coupling. Although both mechanisms affect nonlinear fluid evolution, their mathematical contributions differ substantially.

The analysis demonstrates that Coriolis acceleration acts perpendicular to fluid velocity and therefore contributes no direct work to the standard kinetic-energy balance. Conversely, buoyancy permits energy exchange between kinetic and potential components. In idealised coupled systems, these exchanges cancel within the combined energy balance, while viscosity and scalar diffusion contribute to dissipation. Consequently, energy boundedness depends on the governing assumptions, initial conditions, and dissipative properties of the fluid.

Table 1: Comparative Analysis of Mathematical Properties, Energy Relationships and Stability Mechanisms in Rotating and Stratified Incompressible Fluid Models

Analytical Dimension	Rotating Fluid Models	Stratified Fluid Models	Combined Rotating–Stratified Models
Governing formulation	Navier–Stokes equations incorporating Coriolis acceleration	Boussinesq momentum and buoyancy equations	Coupled equations incorporating rotation and buoyancy
Principal mechanism	Rotational influence on fluid motion	Density-dependent buoyancy forces	Interaction between rotational and buoyancy effects
Major parameters	Angular velocity and Rossby number	Buoyancy frequency, Froude and Richardson numbers	Combined rotational and stratification parameters
Energy relationships	Kinetic-energy balance with viscous dissipation	Kinetic–potential energy exchange	Combined kinetic and buoyancy-energy balance
Wave mechanisms	Inertial waves	Internal gravity waves	Coupled inertia–gravity waves

Dissipation	Viscous diffusion	Viscous and scalar diffusion	Dissipation involving both velocity and buoyancy fields
Solution regularity	Depends on velocity regularity and initial data	Depends on velocity–buoyancy regularity	Requires compatible coupled-field conditions
Stability conditions	Conditional rotational estimates	Conditional buoyancy-related estimates	Coupled stability and instability criteria
Long-term behaviour	Rotation-dependent decay under specific assumptions	Stratification-related decay and dispersion	Coupled perturbation and asymptotic behaviour
Analytical limitations	Unresolved general three-dimensional regularity	Nonlinear coupling and scalar regularity restrictions	Greater interaction complexity and model-dependent conditions

Table 1 demonstrates that rotating and stratified systems possess distinct dispersive characteristics. Rotation generates inertial waves, while stable stratification supports internal gravity waves. Their interaction produces coupled wave modes whose stability depends on rotational intensity, density gradients, and nonlinear flow characteristics.

The comparison further indicates that weak solutions generally provide finite-energy control, whereas strong solutions require additional regularity assumptions for stronger uniqueness and stability conclusions. Similarly, temporal decay and perturbation estimates depend on viscosity, diffusivity, initial-data restrictions, domain geometry, and admissible functional spaces. Therefore, findings obtained under different mathematical conditions cannot be interpreted as universally equivalent.

Combined rotation and stratification may improve particular stability estimates through dispersive interactions. However, nonlinear coupling and background velocity shear can also generate additional instabilities. For example, stable vertical density stratification does not necessarily prevent shear-induced disturbances in rotating flows.

The overall findings establish that energy boundedness, solution regularity, and asymptotic convergence represent different mathematical properties. Finite-energy estimates provide important control over fluid evolution but do not independently establish unconditional global smoothness or long-term convergence. The comparative evidence therefore highlights the importance of model-specific assumptions, compatible

energy formulations, and carefully defined stability criteria when interpreting nonlinear rotating–stratified fluid behaviour.

6. CONCLUSION AND RECOMMENDATIONS

Conclusion

The study has examined the mathematical properties, stability mechanisms, and long-term behaviour of rotating, stratified, and combined rotating–stratified incompressible fluid models through a comparative analysis of established theoretical findings. The analysis has demonstrated that rotational and stratification-related mechanisms influence fluid motion through different mathematical processes. Coriolis forces modify fluid trajectories and generate inertial-wave dispersion, whereas density stratification introduces buoyancy-related restoring forces and internal gravity waves. Their combined influence has highlighted the complexity of nonlinear fluid interactions and the importance of model-specific stability conditions.

The investigation has further established that energy relationships differ across fluid formulations. In rotating systems, Coriolis acceleration contributes no direct work to the standard kinetic-energy balance, while stratified systems permit energy exchange between kinetic and buoyancy-related potential components. Viscosity and scalar diffusivity have been identified as important dissipative mechanisms influencing energy boundedness and solution evolution. The findings have also indicated that weak solutions provide finite-energy control under suitable assumptions, whereas stronger uniqueness and regularity conclusions require additional mathematical conditions.

The comparative assessment has demonstrated that rotational intensity, buoyancy frequency, initial conditions, and nonlinear coupling influence long-term behaviour differently across mathematical models. Although conditional stability and temporal decay results have been established for particular configurations, they cannot be generalised to unrestricted three-dimensional systems. Overall, the study has clarified the relationships between energy estimates, mathematical regularity, and stability while recognising the limitations of existing analytical evidence.

Recommendations

1. Strengthening Comparative Mathematical Analysis of Rotating and Stratified Models

Future investigations should develop consistent comparative frameworks for examining governing equations, mathematical parameters, and stability assumptions. Particular attention should be given to

distinguishing fundamental model properties from conclusions derived under specialised boundary conditions or restricted initial data.

2. Improving the Interpretation of Coriolis and Buoyancy-Related Stability Effects

Researchers should examine rotational dispersion and buoyancy restoring forces as distinct but interacting mechanisms. Greater emphasis should be placed on identifying circumstances in which their coupling suppresses disturbances and conditions under which shear or nonlinear interactions produce additional instabilities.

3. Extending Energy-Based Analysis of Coupled Fluid Systems

Future studies should systematically apply combined kinetic and buoyancy-energy estimates, appropriate dissipation inequalities, and a priori bounds. These approaches should distinguish energy exchange from energy loss and assess boundedness under explicitly defined mathematical assumptions.

4. Advancing Functional-Space-Based Comparisons of Solution Regularity

Further investigations should examine Sobolev, Besov, and related functional spaces to clarify the mathematical requirements governing weak-solution existence, strong-solution regularity, uniqueness, and conditional stability. Comparisons should account for differences in velocity and buoyancy regularity.

5. Enhancing Investigations of Long-Term Behaviour and Asymptotic Stability

Researchers should investigate temporal decay rates, perturbation thresholds, and inertia–gravity wave interactions under varying rotational and stratification conditions. Additional attention should be given to the influence of viscosity, scalar diffusivity, and initial disturbances on asymptotic convergence.

6. Promoting Greater Precision in Mathematical Assumptions and Stability Interpretations

Future mathematical investigations should clearly document fluid-domain geometry, boundary conditions, initial-data restrictions, rotation rates, buoyancy parameters, and admissible regularity classes. Such precision would improve the comparability, reproducibility, and interpretation of mathematical stability findings.

Areas for Further Research

Further research may investigate refined energy inequalities, improved rotation–stratification stability criteria, and alternative functional-space approaches for analysing nonlinear three-dimensional

incompressible systems. Potential extensions include mathematical investigations of large initial-data conditions, variable density stratification, nonuniform rotational effects, anisotropic dissipation, and complex boundary geometries. Additional studies could examine nonlinear coupling mechanisms, conditional regularity estimates, and the relationship between perturbation growth and long-term energy decay. Particular attention may be directed towards fully coupled rotating–stratified Navier–Stokes–Boussinesq models, where Coriolis forces, buoyancy interactions, and viscous processes operate simultaneously. Such investigations may strengthen the mathematical understanding of stability and instability while identifying new analytical approaches to unresolved questions concerning solution regularity and asymptotic behaviour.

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